9. Preparing-Detecting-Measuring-Modeling

This chapter deals with key problems found at boundaries of abstract and laboratory worlds in a way we have only tangentially addressed so far. The grey zone, named Fence, not only helps communicating (mapping) these worlds, it is also the place for further development of quantum theories that are adapted to space-time events. Bridging phenomena involving sensorial elements to abstract theoretical levels there involves a long journey leading to uprooting connections. Quantum technologies have changed many of the foundational concepts used by those scientists that developed quantum theory. Most of the preceding chapters illustrate, in one way or another, such shifts. Most of the classical concept of object dissolves into material elements sustaining quantum states. A key issue is that a new kind of reality is constructed by our research tools. In this chapter, via the analysis of a number of cases, we will try to illustrate the sort of change that is taken place today.

Measuring devices, sources, sinks, shelved chemical compounds...found and eventually designed and produced from scratch as objects relate themselves via changes of quantum states.

The laboratory technology has gone a fantastic up build sensed by changes ranging from the simple spectroscopic instruments permitting *recording* of atomic and molecular spectra to present day quantum technology advances epitomized by nanoscale chemical imaging of a working catalysts (de Smit, et al. Nature, 456, 222 (2008)).

At a Fence implications derived from quantum state changes are confronted to representation of time and space including time scales, duration, location (presence) of laboratory material elements (objects). This is a reality constructed with the help of theoretic concepts; it is not something that you may simply "observe". Basically, there is no need for observers but of experimenters able to change, capture and interpret signals from the surrounding interfaces.

Quantum and classic theoretic frameworks relies on coordinate sets the origin of which is defined with respect to space-time inertial frames so that a clear-cut correspondence is never a transparent endeavor. Care is required to differentiate real space from configuration space. At the Fence one encounters blends of models such as particle-wave, quantum field theories mediating Hilbert to real space together with so-called classic massive objects acting as measuring devices.

Nevertheless, here the fundamental level corresponds to abstract quantum mechanics wherefrom the concept of quantum state occupies a privileged position. To be sure, quantum states are supported by material systems or generated by sources; scattered (absorbed) by sinks, as in the case of electromagnetic radiation. Do not forget that quantum mechanics is about possibilities and not probabilities; that is, the view is from within the physical situation.

Special theory of relativity introduces a fundamental measure of time via signals propagating at a speed that is independent from the state of uniform motion of sources and sinks. The signal corresponds to electromagnetic radiation (light). In vacuum, the speed of light is constant (Cf.Chapt.6). A second element comes from electrodynamics involving the product of wave-length and frequency: $\lambda\nu=c$; both can be modulated but once the value of one of them is determined the value of the other is fixed; their product is a universal constant. The third element originates from quantum physics relating an electromagnetic frequency ν to an energy difference between energy quantum bases states associated to material systems (Bohr relation). These elements allow for the introduction of length and time standards. Thus, by 1990 the length standard was based on accurate frequency measuring the orange-red emission line of ^{86}Kr; this energy difference translated into a frequency is further read as a measure of length $\lambda=c/\nu$; similarly, the best clocks are atomic clocks based, for instance, on the hyperfine interval of an hydrogen atom measured from the ground state. Measurements are interactions translated to information in one way or another; they do not measure eigenvalues but functions based on them (Cf. Axiom 3 Chapt.3).

At the Fence there is access to measured time and space intervals, energy differences, lifetimes. The time and frequency used in the theoretical framework are continuous parameters represented by real number sets. Lifetimes are accommodated in a complex number space. This enumeration of qualities is given to you to help sensing the type of problems theoretical frameworks are confronted to; many aspects concerning them have already been examined.

Besides pure technical aspects, there are social environments (history) sometimes expressed via the so-called scientists' spontaneous philosophy. This one taints the way physical theories are understood. One of the problems confronting the development of quantum physics relates to a view telling that theory would describe material (natural) objects: position, speed, physical properties directly coupled to things in real space. Therefore an interpretation of the theoretical structure became necessary in terms of the particle ideology where one finds statements of the kind: a material system, say one electron is in (or occupies) a specific base state; linear superpositions are reduced or collapsed whenever a measurement takes place; the statement, Schrödinger cat is either alive or dead or in a linear superposition where the cat is both dead and alive does not make sense;

a real object cannot be seen in that manner. And probably, the statement doesn't make sense but for different reasons. For sure, this ideology must be revised, and if it is to be used at all it must be done with caution. Philosophy has been one of domains of the culture in deep crisis all along the twenty century; the old German idealism and naïve materialism/realism are no longer dominant; Wittgenstein, Heidegger, Carnap and others have severely criticized the philosophies of knowledge that are usually claimed to be required to understand QM (for an overview see D'Espagnat: The Veiled Reality).

This chapter addresses the study of some mathematical elements required to get a little bit on the foundations of quantum mechanics. This corresponds to a level 1 (see introduction) where no correspondence to laboratory situations is required. Yet we close the chapter with an inquiry about what is required to move into real laboratory processes. No attempt at completeness is made. The reader is referred to de Oliveira's "Intermediate spectral theory and quantum dynamics" (Birkhäuser, Basel, 2009) as an appropriate resource. Now, let us remind the reader of some key mathematical elements: Quantum states form linear manifolds, where a unitary scalar product is defined; to each vector |g> there corresponds one and only one conjugated transpose, <g|; the symbol <g|h>, is a complex number, standing for the scalar product between two vectors |f> and |g>; <g|g> is a real number.

All along this book, a number of important quantum physical ideas have been introduced; among them, there is the concept of quantum state as being different from the *base states* used to representing them. The quantum state can be probed by another quantum state serving as probe state via interactions to be specified in the following chapters.

Let us now put some logic order in all this; emphasis is put on what permits differentiating base sets from quantum states. To try understanding QM by including those quantum states that will be related to sequences such as events-instruments-recording seems timely. This requires an extended theoretical framework cast in terms of the unfolding of elements that belong to the world of quantum physics as unity not as separate from one another. Acknowledging possible validity boundaries is one task for those working in the field of QM. Here, it is the elementary energy exchange between quantum systems that is quantized; Planck's discovery is essential in this respect. Average energy exchanges can look as continuous processes but elementary exchange is quantized, Bohr relationship holds. In abstract quantum mechanics the state related to a linear superposition must also be counted-by-one; the elements of the superposition add up to a state that differ from the multiplicity of base states. There are responses of the quantum state that express as a whole. Yet by partially resolving the spectral response of some or all of those base states involved (amplitudes different from zero) a partial measurement obtains; Axiom 4 (Cf. Eq.(2.1.3) takes care of this aspect. The state that counts-by-one is a coherent state. There is still too much to uncover to get an understanding of this type of states. We got an appraisal when we examined Bose-Einstein condensate states: a coherent state can interact with another coherent state in a global manner (coherently); this has been experimentally confirmed.

Relating changes in material properties to variations of quantum states seems a natural hypothesis. But there is no answer to the question: How do quantum states change in real world? This is the wrong question. We know how to calculate those changes in Hilbert space and thence go back to the laboratory to experimentally check the results. An essential aspect of science is the comparison of conceptual models with experimental results. Agreement between what theory including computing predicts and experiments is a necessary but not sufficient condition to establish elementary interpretive maps. Quantum mechanics permit calculating all available possibilities. So far, it does not tell us localization emerging as a space-time point at the Fence.

Measurement contains an element of correlated change of quantum states involving measured and measuring systems. A measuring device registers the change; its material constitution must be susceptible to let the experimenter to extract such register and turn into information or to include the system into a more complex arrangement that exploit the change as input data provoking other changes. By reading such record and comparing to a reference, quantitative measurement become possible.

The measured system responds via its spectral structure. The set of energy eigenvalues and the transition amplitudes (integrals) are basic ingredients. The spectral response involves specific sets of energy differences. The identification of a given quantum state boils down to get the set of non-zero amplitudes with their labels. There can be a spectral activation if and only if the amplitude at the relevant root state, i.e. the one where the spectrum originates, happens to be different from zero. For those energy levels that can put up a relative intensity select a subset that might be sufficient to put labels (partially) identifying the system although the detailed quantum state is not (fully) determined. Below, we look at these problems. But, before doing this leat us present allegorically some aspects of quantum states in E&E-10:

E&E-10-1. Transporting quantum states: an allegory

Get hold of a digital camera, set up a fixed scenario and ask three persons to stand up (and not move!). After fixing camera and participants black out the room and save one frame; let now sunlight to enter and save another frame with fixed-position camera. Obviously, looking at the back screen you will see the scene while the preceding frame is black. The first result is obvious; there is no light, the second one seems trivial: the frame shows the scene that is scaled down with respect to the real thing. Light is a quantum system, and its quantum state has been changed by the interaction with the “motif”. The material system sustains a quantum state that would interact with light. Each pixel acts as a scattering center incorporating the quantum properties of the motif. Light is scatter in all directions that are obviously independent from the camera. Whether you are there or not is irrelevant, the motif scatter the light so that a register device at any position will be imprinted. If you move around the source (motif) one is detecting the same motif. If those people belong to the group of acquantaincies an oral description can be done identifying the members. The camera just fix one of those scattered states containing information on the scattering center.

Comparing the motifs you directly see and the one in the frame besides scaling they are pretty much the same. What has happened? We can say that the quantum state accessed by light is transported to and scaled by the device. So, we have the quantum state related to the material system recorded in the camera device. *You do not transport the objects but transport the relevant quantum states.*

This unusually long chapter introduction points to the novelties new generations will be confronted with. In what follows we bring to the fore a selection of cases that hopefully will help us to understand the new developmental stage quantum worls have achieved. Now we leave in the era of Quantum Mechanics taken as a resource to be used in emerging fields such as quantum information, computing…

9.1. Preparing and Recording

The nature of quantum superposition states and how we can sense or “see” them in our world continues to fascinate scientists. A big difficulty in teaching and understanding of QM when coming to measurement theory is that no universally accepted theoretical model exists (e.g. M. Ozawa J.Math.Phys.25 (1984) 79-87; W.M. de Muynck, arXiv:quant-ph/9901010v1; F. Laloë, Am. J.Phys.69(2001) 655-701).

A central problem is in fact posed by the so-called interpretation of QM. Ballentine (Rev.Mod.Phys. 42 (1970) 358-381) discusses two cases: (1) The Copenhagen Interpretation and (2) The Statistical Interpretation.

In so far linear superpositions or quantum states are concerned, the view (1) asserts that a pure state provides a complete and exhaustive description of an individual system (e.g. an electron); the view exposed in (2) asserts that a pure state provides a description of certain statistical properties of an ensemble of similarly prepared systems, but need not provide a complete description of an individual system.

In the early view, there are correspondence rules relating the primitive concept of state and observable to empirical reality. Observables are mapped on to the set of eigenvalues of a particular class of self-adjoint operators (e.g. Hamiltonians).

The individual systems would occupy one and only one base state; the amplitude appearing in the linear superposition in square modulus represents the probability to find one system in a base state when scanning the ensemble.

The reader may find a careful discussion on these issues in Ballentine's original paper (see also his Quantum Mechanics, A modern development, World Scientific, Singapore, 1998). The quotations made above are sufficient for our discussions.

Two sources of uncertainty become apparent in preparation and measurement of a quantum system; they originate precisely from the way the system is prepared (first aspect) and thereafter the interactions leading to the change of quantum state allowing for detection (second aspect). These two aspects have not always been well acknowledged (de Muynck). Furthermore, the standard theory requires that each individual material system, once it is measured, be in one and only one eigenstate so that the amplitude squared is assigned a statistical interpretation. This sort of transition is named as the collapse of the wave function.

The wave function collapse axiom is not acceptable as a universal mechanism (H. Fidder and O.Tapia, Int.J.Quantum Chem.97(2004)670-678 and references therein), it may work for some situations but not for all cases. Moreover, the statistical model is not a compelling feature as the present work shows; (go back to Chapter 8 for a discussion). Thus, we focus on the specifics associated to a measurement theory originated in the present physical model of the quantum state.

The basic element to make sense of a linear superposition is given in Chapter 2 (Axiom 4); it relies on the response to external probes. First, any material system may show a spectral response to external probes; this applies to the 1-system case as well. This makes a difference for a measurement theory because a spectral family implies pairs of energy eigenvalues having a common origin (root base state). Here, there seems to be a problem: the spectra of a given system would contain an infinite number of them as the energy spectrum can be formed from a infinity of base states. But, such is not the case because *only root base states having non-zero amplitude in the linear superposition can put up a response to the probe*. Normalization to unity permits defining relative intensity responses and the ensemble concept may help understanding modulation in intensity but it is not a fundamental element. The observables, to the extent they can be recorded, are the spectral lines; the amplitudes modulate the relative intensities; transition amplitudes cooperate to the intensity of specific lines.

The probe may be selected so that only a fraction of the spectra is probed. Observe that a quantum system can interact in a quantum mechanical fashion only, namely via spectral response. Here lies the bottom line to quantization of material systems.

The spectral response description differs from standard interpretations and can be considered as an element of a third "interpretation" of quantum mechanics. The gathering of probe and measuring system constitute a detector that function as a unity. A larger unit follows after including the measured system. Quantum *base* *states labels* cannot be erased, amplitudes do. Entangled states for instance complete the global system they must be counted by one and not mix up with the base states. Measurement being defined in terms of the measured/measuring base states, entanglement would act as interaction mechanism that measurement finishes by resolving in one way or another. The important element for this type of measurement is the overlap between measuring/ measured spectral responses.

9.1.1. Copenhagen view challenged

The Copenhagen view of QM requires the existence of a classical macroscopic domain in order to explain the measurement process. Heisenberg uncertainty relations appear as the mathematical expression of the complementarity concept, quantifying the mutual disturbance that takes place in a simultaneous measurement of incompatible observables, say $\hat{A}$ and $\hat{B}$; i.e., the operators do not commute (Cf.Chapt.2).

The standard deviations operators $\Delta\hat{A}$ and $\Delta\hat{B}$ can be arranged in the uncertainty relation with respect to a given quantum state $|\psi\rangle$:

$$\Delta_\psi \hat{A} \Delta_\psi \hat{B} = \Delta A\ \Delta B \geq (1/2)\ |\langle[\hat{A},\hat{B}]\rangle| \qquad (1)$$

The symbol $\langle[\hat{A},\hat{B}]\rangle$ is just $\langle\psi|[\hat{A},\hat{B}]|\psi\rangle$, a quantum mechanical average. The inequality is known as Heisenberg-Kennard-Robertson relationship, which has often been interpreted as the mathematical expression of the disturbance following measurement. This type of inequality was derived in Chap.2, Sect.(2.2) and corresponds to the square root of eq.(2.2.3). Ballentine noted that this relationship does not seem to have any bearing on the issue of joint measurement; instead, this relation can be traced to the preparation process of an initial state as it also follows from Chapter 2 analysis. The reason behind this statement lies in the fact that separately measuring each standard deviation, $(\Delta A)^2$ and $(\Delta B)^2$, and making the product, $\sqrt{((\Delta A)^2(\Delta B)^2)} = \Delta A\Delta B$, this relationship can be experimentally tested. Thus, for the momentum-position operators the quantum state prepared as a plane wave, i.e. an eigenstate of the momentum operator, $\Delta\mathbf{p}=0$ so that $\Delta\mathbf{r}$ must be infinite in such a way that the product has a lower bound, namely, $\hbar/2$. From now on we select the direction of the momentum along the x-axis to simplify the discussion.

Including the screen, the possibility to define position and momentum of a particle passing a slit located at the plane x_S is limited by the screen observables uncertainties

$$\Delta z_S \, \Delta p_{zS} \geq \hbar/2 \qquad (2)$$

This equation defines again the system preparation at the slit. For a particle model, the latitudes δz_S and δp_{zS} of particle position and momentum must satisfy Heisenberg inequality, the product being larger or of the order:

$$\delta z \, \delta p_z \approx \hbar/2 \qquad (3)$$

This relation would represent a lower bound for the disturbing effect due to the measuring instrument on the particles. This would cause a post-measurement state of the object that satisfy the uncertainty relation.

Inequality eq.(3) must be distinguished from the relationship satisfied by the standard deviations Δz and Δp_z:

$$\Delta z \, \Delta p_z \geq \hbar/2 \qquad (4)$$

As already noted, the latter inequality concerns the (initial) quantum state preparation. Thus, eq.(4) being an instance of eq.(1) it does not refer to a joint measurement of position and momentum. On the contrary, eq.(3) makes sense if a particle interacts at the slit; therefore, there is no direct connection to a quantum state in this case.

For the two-slit experiment, according to standard QM, strict completion of eq.(2) would make impossible to determine which hole the electron or photon passes through without at the same time disturbing the electrons or photons enough to destroy the interference pattern. This is a puzzling situation in the particle model. Somewhere there is a missing link.

For Muynck, the problem stems from a poor distinction made between different aspects of preparation and measurement. For the Copenhagen view, a measurement is not perceived as a mean to obtain information about the initial state of the system but as a way of preparing the system in some final state; a post-measurement state. The complementarity problem has actually two aspects: preparation and measurement that are not sufficiently distinguished. The concept of measurement disturbance should apply to eq.(2) and (3) while eq.(1) refers to the preparation step. Muynck concludes: "[w]ith no proper distinction between preparation and measurement the Copenhagen interpretation was bound to amalgamate the two forms of complementarity, thus interpreting the Heisenberg-Kennard-Robertson uncertainty as a property of (joint) measurement."

In early quantum mechanics, complementarity distinguishes a world of quantum phenomena from the realms of classical physics; *simultaneous* observation of wave and particle behavior is not allowed by this principle.

Progress in quantum technology permitted Scully et al. (Nature 351 (1991) 111-116) to experimentally investigate such issues by setting up an atom interferometer apparatus. Two questions were put forward in Scully et al. experiments: 1) Whether Heisenberg inequality eq.(3) is relevant for interference experiments; 2) Whether Heisenberg inequality is applicable at all; remember Ballentine's stance that quantum mechanics is silent about the joint measurement of incompatible observables.

The analysis within the particle model interpretation is developed in Scully's paper just referred to. We suggest the reader to carefully study this paper. Here, we quote the result most relevant to the present discussion: "…*we find that the interference fringes disappear once we have which-path information, but we conclude that this disappearance originates in correlations between the measuring apparatus and the system being observed. The principle of complementarity is manifest although the position-momentum uncertainty relation plays no role*."

The interpretation of QM formalism in terms of the complementarity principle leads to puzzling situations; the particle-wave view seems to be fundamentally flawed to the extent classical concepts do not belong to an interpretive framework to quantum mechanics. The question is to know whether a view independent from the particle-wave model permit understanding the experimental results and predict new ones.

9.1.2. View from the Fence

Let us discuss the above phenomenology from the viewpoint presented here where no particles but quantum states and I-frame quantum systems occupy center stage. This means that question such as what path a particle might have followed has not a place.

First, let us state some of the premises again: A system interacts via its quantum spectra; a quantum system can interact in a quantum manner with any other system even if considered as a classical one (remember allegory E&E-10). The receptors function as basic quantum systems at least locally; trivial cases are vision organs; skin heat sensors, heat comes as electromagnetic probes; Planck's discovery applies: elementary energy exchange does it in quanta. Even if an elephant is considered as a macroscopic object, the communication with the world surrounding the animal is made via quantum receptors. The reference to classical systems is just conventional. Observe, however, that for nearly all-practical purposes a classical physics description might be adequate, especially if you want to know where the elephant is heading. The separation between classical/classical is hence a practical custom not a physical law.

Coming back to the two-slit experiment there is a sensitive surface located at a point beyond the two-slit surface acting as a recorder or detecting screen. For atom interference, if we prepare the system to get 1-systems sequentially, the elementary result is an event (click) at a detecting screen (location $x=x_D$); the interaction at this Fence is local. All interactions are local; they involve a change of quantum state where the energy exchange is measurable in terms of energy quanta (electro-magnetic radiation); thus interactions propagate at finite speed.

First, consider the situation that may lead to an event (click) without including it yet. We have to determine the quantum state in a neighborhood of $x=x_D$ surface.

This situation requires that the quantum states entering the description of a given experiment be carefully defined:

i) Before interaction with the double slit;

ii) Quantum state preparation at the slits;

iii) Quantum state beyond x_S that can propagate towards the detecting screen.

Hence the quantum state just before interaction at $x=x_D$ must be well defined. This is a key point.

The *quantum state incident* to the screen with a double slit is usually taken as a plane wave; this is a useful model for a coherent state, the reciprocal vector **k** characterizes the base state. What is the meaning assigned to the initial state? For the time being, it is a laboratory prepared state; the material system sustaining the quantum states may be a molecule, atom, electron or photon. Examine the interactions; the origin for the slits is put on the y-axis. The plane wave propagates along the x-axis so that δp_y is zero by construction and δy must be infinite in such a way that an equivalent to inequality eq.(4) holds; replace z by y only. The quantum state $|\Psi>$ of the I-frame for a massive system is multiplied by the plane wave phase: $\exp(ik_x x)$; the point is that **k** and **x** are vectors indicating direction and position in laboratory space, respectively; the function $\exp(ik_x x)$ is a base function belonging to a linear vector space (rigged Hilbert space); note that there is a double use of the (**k**,**x**)-space, a pointer in real space, on the one hand, and support for the base functions, on the other. The spaces as used in this context are not commensurate! The case selected here for a 1-system the *internal* quantum state $|\Psi>$ is not affected by interactions with screen's holes. Or so the story goes.

The quantum state just before the holes is $C(k_x)\exp(ik_x x)|\Psi\rangle$; it is the same product at the slits also, the amplitude $C(k_x)$ is a fixed number. The quantum state of the system interacting at the holes does it quantum mechanically. As a result, each hole appears as a scattering center. The imprint of the classical object leads to localized emission amplitudes from the slits. Note that it is the initial quantum state that interacts simultaneously with the screen. The base states of interest are those propagating beyond the screen:

$$|\text{Hole-1}\rangle = \exp(i\gamma_1)\langle x,y,z|1\rangle \exp(i(k_x x + k_{y1}(y-D) + k_{z1}z))|\Psi\rangle;$$

$$|\text{Hole-2}\rangle = \exp(i\gamma_2)\langle x,y,z|2\rangle \exp(i(k_x x + k_{y2}(y+D) + k_{z2}z))|\Psi\rangle.$$

The holes centers are located on the y-axis at distance 2D and radius $d \ll D$. To alleviate notation the z-component is taken equal to zero in the Fresnel integrals $\langle x,y,z|1\rangle$ and $\langle x,y,z|2\rangle$. If there are differences in the interaction, the phase γ_1 and γ_2 might differ.

Let the quantum state after screen at x_S be designated as $|\Phi\rangle$. The fundamental principle of QM allows us to write the quantum state as a linear superposition:

$$|\Phi\rangle = C_1\,|\text{Hole-1}\rangle + C_2\,|\text{Hole-2}\rangle =$$

$$\langle\text{Hole-1}|\Phi\rangle\,|\text{Hole-1}\rangle + \langle\text{Hole-2}|\Phi\rangle\,|\text{Hole-2}\rangle$$

Because the case discussed now does not affect the internal state let us focus on the pure space part designated as $\langle x,y,z|(x>x_S)\rangle$:

$$\langle x,y,z|(x>x_S;\ z=0)\rangle =$$

$$C_1\exp(i\gamma_1)\langle x,y|1\rangle\exp(i(k_x x+k_{y1}(y-D)) +$$

$$C_2\exp(i\gamma_2)\langle x,y|2\rangle\exp(i(k_x x + k_{y2}(y+D)) \qquad (5)$$

This is a plane wave state in so far x-axis propagation is concerned. The quantum state includes information about interactions at the slits via amplitudes, phases and Fresnel integrals.

The global quantum state $|\Phi\rangle$ propagates until getting at a neighborhood of a detecting screen. The key result is a well-defined quantum state given by $\langle x,y,z|\Phi\rangle = \langle x,y,z|(x>x_S;\ z=0)\rangle\ |\Phi\rangle$; the (generalized) function eq.(5) is well defined at any point of a screen that can now be replaced by a recording screen thereby allowing for interactions to take place. It is worth emphasizing that eq.(5) requires that the material system sustaining the quantum state went through the double-slit.

Thus, whatever a recording screen will record, this recording must be (co-) related to the interaction with the quantum state generated at the double-slit; this latter being co-related to the source quantum state. Before discussing this part of the problem let us see whether we can glean some information from the values a quantum state has at a screen located at x_D that has not been sensitized yet.

A firm result will be this: if the value of the function at a given neighborhood to a point on the surface is zero, whatever you do there will never be a spectral response derived from a quantum mechanical interaction at that neighborhood. Another one: any finite value different from zero of the quantum state function at a given neighborhood of a point opens the *possibility* for a response from a properly sensitized surface. At any rate, energy must be conserved.

At the level commonly used to discuss a quantum theory of measurement, the actual interaction mechanism between the measured and measuring system, that is responsible for energy exchanges between them, is not explicitly included. Thus, this representation level suggests all possibilities a quantum state has to interact with something else. Why do we not include such effects from the beginning? A reason may be that the founder fathers extracted the basic elements of the process via "thought experiments" where actual material mechanism was not relevant. Here, we proceed along this line; later on more detailed frameworks are described.

Interference effects result from overlap between the *states* originated at each slit. Fresnels' functions have zero overlap just after the double-slit screen. The quantum state in this region would correspond to two separate (non-interacting) beams. Helped by collimators, a two-channel state can be prepared if one want to do this. Because these channels are separated in real space experiments can be designed that will modulate each channel at will; Scully et al. paper present thought experiments using this type of device.

The condition for interference to appear was the *sameness* of the quantum state that interacts at the holes; the holes being of equal nature. When the two-channel state is manipulated so that they come up different from each other and they are channeled to another two-slit device, interference cannot be necessarily expected; the response at a given point is determined by the numeric value of the amplitude. Whenever a mechanism is set up to restore sameness to both channels, then interference effects will show up necessarily.

The mathematical expression of $\langle x,y,z|(x=x_D)\rangle$ yields further insights.

$$\langle x,y,z|(x=x_D)\rangle = C_1\exp(i\gamma_1)\langle x_D,y|1\rangle\exp(i(k_x x_D+k_{y1}(y-D))) + C_2\exp(i\gamma_2)\langle x_D,y|2\rangle\exp(i(k_x x_D+k_{y2}(y+D))) \quad (6)$$

The factor $\exp(ik_x x_D)$ indicates where to look at, namely at the plane surface located at x_D; k_{y1} points in the direction of interest from the frame located at hole-1; k_{y2} does it from hole-2.

Furthermore, look at the state corresponding to y=0 and z=0, the values of the integrals $\langle x_D,y=0\ z=0|1\rangle$ and $\langle x_D, y=0,z=0|2\rangle$ are equal so that:

$$\langle x,y,z|(x_D,y=0,z=0\rangle = \exp(ik_x x_D)\{C_1\langle x_D,y=0|1\rangle\exp(-ik_{y1}D)) + C_2\exp i(\gamma_2-\gamma_1)\langle x_D,y=0|2\rangle \exp(ik_{y2}D)\} \quad (7)$$

The amplitude reflects the geometric characteristic of the holes and the physical character of the interaction cloaked in the coefficients C_1, C_2 and the value of the Fresnel integral.

A response in the intensity regime leads to

$$I(x_D,y=0,z=0) = |\langle x_D, y=0,z=0|1\rangle|^2 \{|C_1|^2+|C_2|^2 + C_1C_2^* \exp(-i(k_{y1}+k_{y2})D) \exp i(\gamma_2-\gamma_1) + C_2C_1^* \exp(i(k_{y1}+k_{y2})D) \exp{-i}(\gamma_2-\gamma_1) \quad (8)$$

For this specific set up, $(k_{y1}+k_{y2})=0$ and if no phase effects were introduced by the interaction, namely, $(\gamma_2-\gamma_1)=0$, the intensity depends upon the amplitudes:

$$I(x_D,y=0,z=0) = \langle x_D, y=0,z=0|1\rangle\langle x_D, y=0,z=0|2\rangle \{|C_1|^2+|C_2|^2 + C_1 C_2^* + C_2 C_1^* \} \quad (9)$$

The cross terms add up to the real part of the product: $2\mathrm{Re}\ C_1C_2^*$. The intensity at this point reflects the nature of the interactions between the ingoing quantum state and the 2-slit device. The particular case $C_1 = C_2 = 1/\surd 2$ and $(\gamma_2-\gamma_1)\neq 0$ produces the result:

$$I(x_D,y=0,z=0) = |\langle x_D, y=0,z=0|1\rangle|^2 \{ 1 + 1/2(\exp i(\gamma_2-\gamma_1) + \exp{-i}(\gamma_2-\gamma_1)\} \quad (10)$$

An interesting case corresponds to $(\gamma_2-\gamma_1)= \pi/2$. For now, the real part in the interference term $\cos(\gamma_2-\gamma_1)$ annihilates while the imaginary part of the exponentials sum cancels out. The intensity being $|\langle x_D,z=0|1\rangle|^2$ while for $(\gamma_2-\gamma_1)=0$ the intensity is twice this value.

E&E-9.1-1Check with the amplitudes the above results

The look at the amplitudes is actually the thing to do to check consistency. From eq.(7) we have:

$$|(x_D,y=0,z=0\rangle=$$

$$\exp(ik_x x_D)\ \{C_1\langle x_D,y=0|1\rangle\exp(-ik_{y1}D)) +$$

$$C_2\exp i(\gamma_2-\gamma_1)\langle x_D,y=0|2\rangle\ \exp(ik_{y2}D)\} =$$

$$\langle x_D,\ y=0,z=0|1\rangle\exp(ik_x x_D)\ \{C_1\exp(-ik_{y1}D)) +$$

$$C_2\ \exp(ik_{y2}D)\exp i(\gamma_2-\gamma_1)\}$$

At this point the geometric set up yields $\exp(-ik_{y1}D)= \exp(ik_{y2}D)$ because $k_{y1}=-k_{y2}= k_{yD}$ a fixed value:

$$|(x_D,y=0,z=0\rangle= \langle x_D,\ y=0,z=0|1\rangle\exp(ik_x x_D)\ \exp(-ik_{yD}D)\ \{C_1 + C_2\ \exp i(\gamma_2-\gamma_1)\} = \langle x_D,y=0,z=0|1\rangle\exp(ik_x x_D)\ \exp(ik_{yD}D)$$

$$\times\ \{C_1 + C_2\ (\cos(\gamma_2-\gamma_1)+i\sin(\gamma_2-\gamma_1))\}$$

For $(\gamma_2-\gamma_1)=\pi/2$ and $C_1 = C_2 = 1/\sqrt{2}$ the amplitude reads:

$$|(x_D,y=0,z=0\rangle = \langle x_D,y=0,z=0|1\rangle\ \exp(ik_x x_D)\ \exp(ik_{yD}D)$$

$$\times\ (C_1 + iC_2)$$

The amplitude is now a complex number and the intensity response depends on $|\langle x_D,z=0|1\rangle|^2$ that is multiplied by $(C_1 +iC_2)\ (C_1 -iC_2) = |C_1|^2+|C_2|^2 =1$. The wave function at this particular point is normalized to unity, as it should.

For an interaction at the slits producing a difference in phase $(\gamma_2-\gamma_1)=\pi$ it is easy to check that the intensity at the origin of the recording screen defined here annihilates.

The preceding discussions show that the amplitude at the mid-point between the slits can be modulated in different manners and this modulation is reflected at the screen where we may put a detector device. The interactions at the 2-slit determine the type of response to be measured at the recording screen. The fringes may disappear if the amplitudes C_1 and C_2 are independently changed thereby breaking the coherence.

The correlations do not originate between the measuring apparatus and the system being observed; the quantum state at the recording screen is totally independent from the presence of such a sensitized screen. This is a fundamental theoretical result: *It is measurement of the wave function, which is the basic element of the theory.*

The event showing the local interaction appears to be contingent in so far place and time are concerned. But it is the necessary correlate to the interaction between both systems: screen and quantum states of the material system we are interested in measuring. It is the change in quantum state elicited by the event that put in evidence a presence, namely, that of the material system supporting the quantum state. But we are not yet there with the theoretical description.

We also have the shadows corresponding to the slits we put on the first screen; here for shadow-1 the wave vector component will be k_{yD1} and k_{zD1} for a circular slit; to get a simple picture we keep z=0.

If we look at the intensity pattern we can infer a simple result, namely, a decrease in amplitude squared from the origin in the detection surface towards the origin of the slit's shadows. For the intensity pattern this is controlled by the overlap between the Fresnel integrals. But there is need for a reference just in case we decide to close one slit for example.

The quantum state interacting with the open one-slit is the same as the one we used with the open two-slits. The interaction generates a scattered state represented by eq.(6) by taking C_2=0 to get:

$$\langle x,y,z|(x=x_D)\rangle = C_1\exp(i\gamma_1)\langle x_D,y|1\rangle \times \exp(i(k_x x_D + k_{y1}(y-D)).$$

To this base state adds the incoming initial state $\exp(ik_x x)$ to form a diffraction quantum state:

$$\langle x,y,z|(x=x_D);\text{diffraction}\rangle = \exp(ik_x x) \times \{1+ C_1\exp(i\gamma_1) \langle x_D,y|1\rangle\exp(i(k_{y1}(y-D)\}$$

The intensity pattern obtains from $|\langle x,y,z|(x=x_D);\text{diffraction}\rangle|^2$. The contribution to the diffraction takes on the simple form when $C_1 \langle x_D,y|1\rangle$ is taken as the product of two real numbers: $2C_1 \langle x_D,y|1\rangle \cos(f(x,y)) + C_1^2 \langle x_D,y|1\rangle^2$; here f(x,y) is the exponential argument, $(k_x x+ \gamma_1+ k_{y1}(y-D))$.

Consider a Fraunhoffer diffraction case. At the detecting surface there will be a maximum in intensity at the center of the shadow-slit. Followed by concentric circles with decreasing intensity. The diffraction pattern will reproduce itself at shadow-slit 2 when slit-1 is closed. If there were no interference between states 1 and 2 one would expect a sum of each slit diffraction intensity pattern. Thus, very little intensity in between the shadow slits.

Two-slits being simultaneously open and $(\gamma_2-\gamma_1)=0$: The preceding analysis shows that the intensity of the diffraction pattern at a shadow-slit becomes weighted down by the small overlap Fresnel integral. However, the intensity increases at the middle. The interference pattern would clearly appear. That's it. The complete diffraction pattern should appear if we could measure the quantum state or something related to.

E&E-9.1-2 Scattering theory perspective

The diffraction quantum state can be seen as a specialized form of the quantum state obtained from scattering theory. The initial state for the model considered above is the plane wave in the direction indicated by the wave vector **k**: $\exp(i\mathbf{k}\cdot\mathbf{R})$. The initial state is then $\exp(i\mathbf{k}\cdot\mathbf{R})|\Psi\rangle$. The interaction at the slit does not change the internal I-frame quantum state and the outgoing state $|g+\rangle$ projected in real space coordinates $\langle\mathbf{R}|g+\rangle = \Phi_g^+(\mathbf{R})$ is given approximately by:

$$\Phi_g^+(\mathbf{R}) \approx [\exp(i\mathbf{k}\cdot\mathbf{R}) + (\exp(i|\mathbf{k}|\,|\mathbf{R}|)\,/|\mathbf{R}|\,)\, C(E, \mathbf{k}\,/|\mathbf{k}|, \mathbf{k}_c\,/|\mathbf{k}_c|\,)]\,|\Psi\rangle$$

The unit vector $\mathbf{k}_c\,/|\mathbf{k}_c|$ indicates the direction of the scattered state, $\mathbf{k}/|\mathbf{k}|$ is the unit vector indicating the direction of the vector **k**. The amplitude $C(E, \mathbf{k}\,/|\mathbf{k}|, \mathbf{k}_c\,/|\mathbf{k}_c|)$ has dimension of length due to the form assigned to the scattered state. The differential cross section $\sigma(E,\mathbf{k}\,/|\mathbf{k}|,\mathbf{k}_c/|\mathbf{k}_c|)$ is proportional to $|\,C(E, \mathbf{k}\,/|\mathbf{k}|, \mathbf{k}_c\,/|\mathbf{k}_c|)\,|^2$.

For the problem we have been discussing take square modulus of the amplitude in square brackets. There is need to calculate $\exp(i\mathbf{k}\cdot\mathbf{R})$ explicitly.

$$\exp(i\mathbf{k}\cdot\mathbf{R}) = 4\pi\, \Sigma_{\ell=0,\infty}\, \Sigma_{m=-\ell,+\ell}\; i^{\ell}\; Y_{\ell,m}^{*}(\mathbf{k}\,/|\mathbf{k}|)\, Y_{\ell,m}\,(\mathbf{k}_c/|\mathbf{k}_c|)$$
$$\times\, j_\ell(|\mathbf{k}|\,|\mathbf{R}|)$$

The $Y_{\ell,m}$ is a spherical harmonic (base function of the angular momentum $\hat{L}^2$ (Cf. Sect.3.5), the function $j_\ell(|\mathbf{k}|\,|\mathbf{R}|)$ is a spherical Bessel function of order ℓ. A similar expansion must be carried out for the amplitude function. Name the unit vectors as $\mathbf{n}_\mathbf{k} = \mathbf{k}\,/|\mathbf{k}|$ and $\mathbf{n}_{\mathbf{k}c} = \mathbf{k}_c/|\mathbf{k}_c|$ the amplitude of the scattered state reads:

$$C(E, \mathbf{n}, \mathbf{n}_c) = (2\pi i/k) \Sigma_{\ell=0,\infty} \Sigma_{\square=-\ell,+\ell}\ i^{\ell}\ Y_{\ell,m}{}^{*}(\mathbf{n_k})^{\ell}$$

$$\times Y_{\ell,m}(\mathbf{n_{kc}})\ T_{\ell}(E)$$

Where $T_\ell(E)$ is a phase factor: 1-exp($2i\phi_\ell$). The phase shift ϕ_ℓ is due to the interaction with the slit.

The function $\Phi_g^+(\mathbf{R})$ represents in this context our |(x=x_D); diffraction>. The assumption is that the scatter center is spherical symmetric.

9.1.3. Quantum states and recording screens

The quantum state discussed above not only must have response spectra able to disclose the interaction with the measured quantum system but also must let register it. At this point the Fence makes its appearance. If no-register is implied, quantum interactions entangle measuring/measured quantum states keeping everything in Hilbert space. You will never get funded to produce Hilbert space knowledge only! At some point information must be *produced*; this implies a change of entropy (energy dispersion). Even if we can claim that it is the evolution in Hilbert space that determines the way the material system sustaining the quantum state (thing) will change it is in the laboratory world with its objects in real space that such effects should manifest.

Information production has at least two aspects: 1) an imprint in a material support; 2) an interpretation (reading) of such imprint. The first item concerns physics; the second aspect has a social aspect that at the very least expresses itself via communication channels. To make things simple: the photon field impinging your eyes may produce a record, if you have never seen or informed about the shape of the object reflecting light out there the response of yours might certainly be of surprise if not panic, and if asked to describe what can it be, no words might be available. You have learned about the things in the world we live in. Thus, an interference pattern is a well-defined imprint that we have learned to control and use it as an instrument. In this book we stop most of the time at the physical level: we are studying, preparing, changing or using as devices what we have called quantum states.

So much for the introductory aspects: Let us consider some examples of how information can be produced. Let us first propose a toy model that can help us in this respect.

Consider a two-state I-frame quantum system. The ground base state |0> and |1> an excited state gathered in the column vector [|0> |1>] are used to represent any quantum state of this quantum reading system; the energy difference $\Delta E = E_1 - E_0$, a positive quantity, can be radiated with a frequency ν such that $\Delta E = E_1 - E_0 = h\nu = \hbar\omega$.

The quantum state (C(0) C(1)) for which C(0)=1 and C(1)=0 corresponds to the ground state: |1,0> = (1 0). Off-resonance frequencies ω' put the system in a linear superposition state with amplitudes proportional to (cos ω't sin ω't); the I-frame system is coupled to the electromagnetic field.

Consider a resonance case: ω'= ω . For the global system there are two states: $|0>|n_\omega=1>$ and $|1>|n_\omega=0>$ with equal energy; note the qualitative difference: for $|0>|n_\omega=1>$ the energy is in the field; while $|1>|n_\omega=0>$, energy localizes at the material system the photon field has not available energy (it is "empty").

But now you know something about this system. For example, if the amplitude at base state $|1>|n_\omega=0>$ is near one, the interaction with the vacuum prompts for the spontaneous emission in any arbitrary direction; the I-frame system "return" to the ground state: $C(0,n_\omega=1) \approx 1$ while $C(1,n_\omega=0) \approx 0$. We do not want this because a full control of the energy is required to construct a theoretic recorder; thus, select adequate material systems to sustain this type of states with minimal spontaneous emission. Assume we have it. A premise required for a quantum system to be transformed into a detector is laid down. For a finite time interval one can consider the energy localized at the I-frame system. By connecting this I-frame to a secondary system able to catch up that energy and use it in activating a cascade process we have an event. The catching up event can be amplified and stored somewhere else; the actual mechanism of recording may well enter into complex devices that are not the focus for the elementary measurement model we are discussing. An important point is that the recording subsystem is left in a ground state, probably ready to act again. More fundamental is the local character of such event borrowed from the local nature of the I-frame.

A new, fundamental feature has pop up: the event leading to an actual energy transfer between the measuring I-frame system and the amplifier device. This kind of event discloses an interaction with the measured system albeit indirectly. At any rate, energy conservation is required because this is a Fence event between two different material systems.

To remain in Hilbert space implies that the measuring/measured quantum systems remain in an entangled state unknown to people at the Fence. But all *possible* changes are there anyway. To disclose them, energy must be exchanged and consequently entropy must vary. One is coming close to thermodynamics as soon as the *description* of phenomena forces the systems to leave Hilbert space.

So far a photon field has represented the Fence quantum system of interest. We do this not only because it is easier to construct the formalism but also EM radiation is an energy carrier allowing for sensing the sources of such radiation. The photon field transports the information about an external (to Hilbert space) world; you do not have an elephant at the retina but a coded quantum state; the information is there to be decoded, we call it an image and society assign a name (label) to that wich is the case. Thus, the material system sustaining the quantum states must have energy to exchange with the I-frame detector system or the measuring field can be the energy source.

Retain the two-state model idea for the photon field and define the operator: $\hat{\aleph} = |-><-| + |+><+|$ as the unit operator in this new abstract space. The energy quantum state can be written as a linear superposition:

$$|\Delta E> = |-> <-|\Delta E> + |+> <+|\Delta E>$$

Let prepare the system in the particular state $<+|\Delta E>=1$ and $<-|\Delta E>=0$. This means that the system can deliver an energy ΔE in the transition $|+> \rightarrow |->$. The energy states are used in conjunction with the scattered quantum state.

The total wave function after interaction with the double slit including energy states is given by:

$$<x,y,z|(x>x_S)> |\Delta E> = \{C_1 \exp(i\gamma_1) <x,y|1> \exp(i(k_x x + k_{y1}(y-D))) + C_2 \exp(i\gamma_2)<x,y|2> \times \exp(i(k_x x - k_{y2}(y+D)))\} |\Delta E> \quad (9.1.3.1)$$

The operator $\hat{\aleph}$ permits introducing a possible energy exchange into the abstract formalism in a simple manner. Using the I-frame energy basis, the quantum state can be written as:

$$<x,y,z|(x>x_S)> |\Delta E> =$$
$$C_1 \exp(i\gamma_1)<x,y|1> \exp(i(k_x x + k_{y1}(y-D)))$$
$$\times\ (|-> <-|\Delta E> + |+> <+|\Delta E>) +$$
$$C_2 \exp(i\gamma_2)<x,y|2> \exp(i(k_x x - k_{y2}(y+D)))$$
$$\times (|-> <-|\Delta E> + |+> <+|\Delta E>) \quad (9.1.3.2)$$

Reordering this equation to get the amplitudes acting at the base state for energy exchange and defining terms:

$$A(C_1,C_2,\gamma) = \{C_1\exp(i\gamma_1)<x,y|1>\exp(i(k_x x+k_{y1}(y-D))) + C_2\exp(i\gamma_2)<x,y|2>\exp(i(k_x x-k_{y2}(y+D)))\}$$

one gets:

$$<x,y,z|(x>x_S)>|\Delta E> =$$
$$A(C_1,C_2,\gamma)\ \{<-|\Delta E>|->+<+|\Delta E>|+>\} \qquad (9.1.3.3)$$

From the perspective of the photon field the intercation information embedded in the function $A(C_1,C_2,\gamma)$ imprints in both channels. Now, look at the quantum state for an I-frame system prepared in the state $<+|\Delta E>=1$ and $<-|\Delta E>=0$ the propagating quantum state is the linear superposition:

$$\{C_1\exp(i\gamma_1)<x,y|1>\exp(i(k_x x+k_{y1}(y-D)))+$$
$$C_2\exp(i\gamma_2)<x,y|2>\exp(i(k_x x-k_{y2}(y+D)))\}|+>\}$$

The component $|->$ has zero amplitude.

The response in intensity at the detecting screen is just equation:

$$|\{C_1\exp(i\gamma_1)<x,y|1>\exp(i(k_x x+k_{y1}(y-D)))+$$
$$C_2\exp(i\gamma_2)<x,y|2>\exp(i(k_x x-k_{y2}(y+D)))\}|^2 \qquad (9.1.3.4)$$

This simply means that ***at whatever space point the amplification event might take place, the system would be measuring the whole interference pattern at given neighborhood***. A local detector permits calculating the amplitude just there so that the amplifier may or may not be triggered.

The actual localization of a triggering process does not belong to the present model. Once it happens, a spot (click) becomes visible (localized). The spot produced by the material I-frame system is given a particulate property. However, there is no compelling quantum mechanical reason that would permit to identify the real space event to a particle, although in the particle model philosophy such assignment would seem natural. We are bordering here issues related to contingency and necessity. These issues are left to the following subsections after some experimental input on diffraction/interference experiment is introduced to you.

9.1.4. Events at recording screen

The feature we have just found at the Fence is the activation of a detection screen. It appears as an event localized in real space that can be timed once the system is clocked when it was set up in action. Thus, an arrival time can be registered. All these events occur then in real (laboratory) space.

There is more. Tonomura reported experiments on electron interference using field-emission electron microscope equipped with an electron biprism and 2-dimensional position-sensitive electron-counting system (Ann.Acad.Sci.NY.755 (1995)227-240). Electron events could be counted one by one on a TV monitor. Let us describe some results from the present perspective.

1) The first event may happen anywhere on the TV screen; you can prepare the system as many times as you want and check that the first event appears localized (almost) at random; this randomness is only apparent if you use the theory presented here. What has happened was a change in amplitudes for a transition from state |+> to |-> by capturing energy from the I-frame system; the relative intensity being:

$$|\{C_1\exp(i\gamma_1)\langle x,y|1\rangle\exp(i(k_x x+k_{y1}(y-D))) +$$

$$C_2\exp(i\gamma_2)\langle x,y|2\rangle \exp(i(k_x x- k_{y2}(y+D)))\}|^2 \quad (9.1.4.1)$$

This is calculated at the position of the spot. Thus, the information contained in the spot is there to reflect the value given by the mathematical framework; we know then that it makes part of an interference pattern. Why does it appear at that position and not at another one? Such a question is related to the amplifier device not to the quantum mechanics behind.

2) When the number of electrons from the source increases an interference pattern becomes recognizable. This result elicits eq.(9.1.4.1).

3) "*Even when the electron arrival rate is as low as 10 electrons/s over the entire field of view (so that there is at most only one electron in the apparatus at one time), the accumulation of single electrons still forms the interference pattern.*" This statement made by Tonomura can be seen from the present perspective in a different light. The emergence of an interference pattern for the present quantum mechanical description is ensured even if you send one-by-one during a period of one year or a millennium (all other things been equal) provided the information revealed by the spot is directly related to the values of eq. (9.1.4.1). The

distribution when there are few electrons may *seem* quite random but the what is *emerging* there is just the quantum state given as an amplitude squared in eq.(9.1.4.1).

4) Even if there are two slits there has never been talk about which slit (path) the electron has taken. Quantum mechanics is about quantum states. If you know which slit is used then this would mean that either C_1 or C_2 is zero. In that case, the interference trivially vanishes. The quantum state has changed because the experiment is different compared to the preceding case.

5) Theoretically there are nodal strips so that no particle spots would be appearing on screen. In practice the screen will end up fully covered if you let the experiment proceed for a long time. You can discuss this result in the classroom! In such situation we would have been doing the theory based on the recording-reading data elicited by the spot counting. The point is that we seek after a quantum mechanical description of the double slit diffraction from abstract Hilbert space quantum mechanics. Spot counting occurs in real space with all experimental errors naturally associated to them.

6) There is never talk about collapse of the wave function. Only a quantum transition is involved the wave function is the amplitude affecting the base state |+>. With hindsight, the initial state could be restored.

7) Performing a phase change so that $(\gamma_2\text{-}\gamma_1)= \pi$ we know that instead of a maximum in the interference-pattern at the middle there must by a nodal region. The experiment was performed by Tonomura to find just this result.

The energy transfer in our view does not destroy the I-frame quantum state. Only the amplitudes have changed. By disentangling the classical mechanics view of particle from the quantum state sustained by the material system, the “welchen-weg” (which path) problem vanishes.

Yes, the I-frame as a material system must have gone through one of the slits, mustn’t it? But this is not the point. The material system evolves in a real space and the event amplified is as such totally irrelevant to quantum mechanics we are probing, because it tells nothing new; it confirms that quantum state scattering, prepared the way it was, produce a dispersion of the material system that is tightly associated to a specific interference pattern. However, the event is relevant to the experimental work that must be designed so that timing is exactly measured.

A quantum mechanical formalization of time observables, as the one seen in the arrival time above, is still and open and challenging fundamental question (Muga, J.G. and Leavens, C.R. Phys.Rep. 338 (2000) 353-438). The issue has thoroughly been discussed in Muga and Leavens paper. The severe criticism raised by W.Pauli

(General Principles of Quantum Mechanics, Springer-Verlag, Berlin 1980, footnote on page 63) is still valid. The time used in the time ket |t> is just a label, a real number not a dynamic variable; there is nothing like the eigenvalues equation: $\hat{t}$ |t> = t |t> such that $\hat{t}$ commutes with a Hamiltonian operator $\hat{H}$ that is bounded from below. Time scales are defined with the help of quantum *transition* energy, i.e. energy differences, while the energy scale comes from the energy eigenvalues of a bounded Hamiltonian. The transition is sensed by an electromagnetic field. Moreover, it cannot have escaped your attention that time was measured at the moment of I-frame production and time of arrival is conventionally taken as the one recorded for an event. All timing that has happened has done it in real space without "time-spectra". One of the difficulties to overcome is as follow: the eigen values of energy should range from plus to minus infinity as it is for space base kets, time and energy are canonical conjugate, this implies that for Hamiltonians bounded from below, energy cannot range from minus to plus infinity as it should. Relativistic physics does not help; the base states for "antiparticles" must be included with positive energies that are bounded from below (Cf.Chapt.7).

9.2. Preparation and time evolution

Whenever a quantum system includes measured and measuring base states there must be a way to prepare both subsystems independently. Also, the quantum state of the measuring system must be accessible to devices able to read the changes imposed by the interaction. The measuring device and the measured system should not end up in entangled states in order to qualify as a semi-classical apparatus. We examine here some aspects of this problem.

The operator $\hat{H}_D$ defines the measuring material system Hamiltonian (detector). The base state vectors $\{|k^D>\}$ fulfill the equations:

$$\hat{H}_D|k^D> = E_{kD}|k^D>;$$

E_{kD} is the k_D-th energy eigenvalue. For the measured system we have:

$$\hat{H}_S|j^S> = E_{jS}|j^S>;$$

E_{jS} is the j_S-th energy eigenvalue.

The base set for the compound system is the direct product $\{|j^S>\} \otimes \{|k^D>\} = \{|j^S> |k^D>\}$. This separability hypothesis is what makes a detector capable of performing measurements on the object system. The reason is simple: before and after the quantum interaction the detector and object systems should not appear entangled. The quantum state before interaction can be factored as $|\varphi_{(S)},t=t_o\rangle$ $|\phi_{(D)},t=t_o\rangle$ and after the measuring interaction has been accomplished one should have a simple product again: $|\varphi_{(S)},t=t_\infty\rangle$ $|\phi_{(D)},t=t_\infty\rangle$. Thus,

$$|\varphi_{(S)},t=t_o\rangle\,|\phi_{(D)},t=t_o\rangle \leftrightarrow$$
$$\{\Sigma_{j,k}\, C_{j,k}(\tau)|j^S k^D\rangle\}_\tau \leftrightarrow$$
$$|\varphi_{(S)},t=t_\infty\rangle\,|\phi_{(D)},t=t_\infty\rangle \qquad (9.2.1)$$

We have used the Greek letter τ to signal the evolution in Hilbert space. Entangled states are those having amplitudes $C_{j,k}(\tau)$ that cannot be expressed as a simple product, e.g. $C_{j,k}(\tau) \neq a_j(\tau)b_k(\tau)$; in what follows the model requires non-entangled states after interaction between subsystems. The ingoing and outgoing product states can easily be measured at the Fence. Each one has associated an I-frame, and can hence be localized in space. The time measured at the Fence presents no big problems if we can identify the response from the simple product states.

The material content of the measuring and measured systems is invariant. At the Fence this model does not allow for charge transfer effects, only energy exchanges.

Quite a different situation obtains at the interaction zone where the non-interacting systems $\hat{H}_S + \hat{H}_D = \hat{H}_o$ and the interaction term $\hat{H}_{S\text{-}D}$ makes up for the total Hamiltonian $\hat{H}$:

$$\hat{H} = \hat{H}_o + \hat{H}_{S\text{-}D} \qquad (9.2.2)$$

The Hamiltonian $\hat{H}$ stands for a material system whose content is the sum of the material contents of our S- and D- systems. The partitioning given at the right-hand is one among many from a set of n (= $n_S + n_D$) electrons and N (=$N_S + N_D$) nuclei global system. The nuclear charge and the electron charge are conserved; the former are distinguishable, the latter are not. In the relation (9.2.1) the quantum states of the system goes through an enlarged base system corresponding to the base state of $\hat{H}$.

The model chosen for a measuring device implies that the map (9.2.1) holds. In Hilbert space evolution, anything compatible with the spectra of the total Hamiltonian is allowed. The constraint expresses at the Fence, where (for this model) only simple products state amplitudes can be sensed.

The passage via Hilbert space opens all sorts of mechanistic proposals, including charge transfer states in which case such base states must be included from the very beginning; at the Fence energy must be conserved.

Observe that the spectra of the measuring apparatus must overlap the spectra of the measured system, totally or in part. Another fundamental hypothesis, valid for any measurement theory at the Fence, is that spontaneous emission is a negligible phenomenon. Thus, a quantum that is used to change the state from k to k' in the detector comes from a transition j to j' in the measured system; a perfect measuring device must respond to all possible transitions in the measured system. In this case, an imprint of the measured system is to be found in the measuring device. In between the preparation time and the actual recording the global system evolves in Hilbert space. This is a characteristic of Fence processes we already met in the preceding section.

The recording aspect of measurement is represented by an interaction between two quantum subsystems: the measured system given by the Hamiltonian $\hat{H}_S$ and measuring system by the Hamiltonian $\hat{H}_D$. The interaction operator $\hat{H}_{S\text{-}D}(t)$ is a time dependent operator that in units of $h/2\pi = \hbar = 1$ related to the time independent operator $\hat{H}_{S\text{-}D}$ by the ansatz derived from the interaction representation already familiar to you:

$$\hat{H}_{S\text{-}D}(t) = \exp(i\,\hat{H}_o t)\ \hat{H}_{S\text{-}D} \exp(-i\,\hat{H}_o t) \qquad (9.2.3)$$

H_S defines the object system (S) under investigation and H_D characterizes the detector (D).

In the model retained here, the time independent operator is written as:

$$\hat{H}_{S\text{-}D} = (1/2)\,\hat{V}_{S(D)} + (1/2)\ \hat{V}_{D(S)}. \qquad (9.2.4)$$

The operators $\hat{V}_{S(D)}$ and $\hat{V}_{D(S)}$ are formally equal, but arranged differently. Thus, $\hat{V}_{S(D)}$ is interpreted as a field operator acting from system S with a field operator on to the detector D, and vice versa.

As an example take the dipole-dipole approximation, operator $\hat{V}_{S(D)}$ stands for the scalar product $-\vec{\mu}_S.\vec{E}_D$, and $\hat{V}_{D(S)}$ is analogously defined as $-\vec{\mu}_D.\vec{E}_S$; for the case of a Stern-Gerlach experiment, the field $\vec{E}$ stands for a magnetic field and $\vec{\mu}$ is a magnetic dipole operator. Note that the basis set should contain all quantum states of the complete system and, consequently, only transitions among quantum states are forming the kernel used to construct the response functions associated with the subsystems.

Thus, at time t_o one assumes that the object and detector systems are *prepared* in the quantum states $|\varphi_{(S)}>$ and $|\phi_{(D)}>$, respectively; this is equivalent to give a set of initial amplitudes for both subsystems. In Hilbert space, the state prepared at t_o is the product $|\psi(t_o)> = |\phi_{(D)}> |\varphi_{(S)}>$. The time ordering operator $\hat{T}$ allows writing the time evolution of the global system $|\psi(t)>$ as:

$$|\psi(t_f)> = U(t_f,t_o)\ |\phi_{(D)}>\ |\varphi_{(S)}> =$$

$$\hat{T}\{\exp[-(i/\hbar)\int_{t_o}^{t_f} dt' \; \hat{H}_{S\text{-}D}(t')\} \,|\phi_{(D)}\rangle\, |\varphi_{(S)}\rangle =$$
$$\hat{T}\{\exp[-(i/\hbar)\int_{t_o}^{t_f} dt' \; \hat{H}_{S\text{-}D}(t')\} \textstyle\sum_j\sum_k a_j b_k |j^S\rangle |k^D\rangle =$$
$$\Sigma_j\Sigma_k\Sigma_{j'}\Sigma_{k'}\; a_j(t_o)b_k(t_o)\, U^{kk'}_{jj'}(t_f, t_o)|j'^S\rangle|k'^D\rangle \qquad (9.2.5)$$

The initial conditions are $U^{kk'}_{jj'}(t_o,t_o)=\delta_{jj'}\;\delta_{kk'}$ meaning that the evolution operator is the unit operator at initial time.

To read this equation consider first the sums $\Sigma_j\Sigma_k$ that concerns the amplitudes at initial time $a_j(t_o)b_k(t_o)$ that multiply the matrix elements of the evolution operator $U^{kk'}_{jj'}(t_f,t_o)$ for fixed j',k' labels. Thus, for a given final time t_f, this matrix element corresponds to the transition amplitude for the concerted transitions k→k' and j→j' cumulated for time t_f-t_o via all possible mechanisms compatible with the symmetry rules of Hilbert space. If this matrix element were zero, the amplitudes $a_{j'}(t_f)\, b_{k'}(t_f)$ will be zero. Let look at this point in more detail in order to be able to check those amplitudes that differ from the initial ones.

The point is to rearrange the last term in eq.(9.2.5) to make the initial base $|j^S\rangle$ $|k^D\rangle$ appear in the right order. In doing so, comparisons between initial and final states can easily be done. The completeness of the basis permits a recasting of the four dummy indexes and the expression (9.2.5) can be rewritten as

$$|\psi(t_f)\rangle = \Sigma_j\Sigma_k\,(\Sigma_{j'}\;\Sigma_{k'}\;a_{j'}b_{k'}\;U^{k'k}_{j'j}(t_f, t_o)\,)\,|j^S\rangle|k^D\rangle =$$
$$\Sigma_j\Sigma_k\; a_j(t_f)\; b_k(t_f)\,|j^S\rangle|k^D\rangle \qquad (9.2.6)$$

The result can be seen as a rotation of the state vector provoked by interactions (the base states are kept fixed). This result is rigorous provided entanglement can be neglected at the recording level. The case when entanglement is important will be discussed later on.

The full interaction produces changes in quantum states as time progresses, reflected as just numerical changes in the complex coefficients:

$$\{a_j\, b_k\}=\{a_j\,(t_0)b_k(t_0)\} \rightarrow$$
$$\{a'_j b'_k\} = \{\, a_j(t_f)\; b_k(t_f)\}. \qquad (9.2.7)$$

In the standard measurement theory, the matrix elements $U^{kk'}_{jj'}$ are obtained by applying the von Neumann operator, defined by Zeh, H. D. (Found.Phys. 1970, 1, 69-76) as:

$$U(vN)^{kk'}_{jj'} = \delta_{jj'}\; U^{kk'}_{jj'}\,. \qquad (9.2.8)$$

The idea behind this latter equation is that the object system is put into a stationary state (actually a base state) by the measurement and remains there henceforth. As a consequence, it states that consecutive measurements of the same state return the same eigenvalue. This model is not retained in our approach; from the perspective developed in this book such hypothesis is not even wrong, it is simply non-commensurate.

On the contrary, what can be measured in our approach is the set characterizing the transitions between the base states (stationary states), e.g. Balmer's series of the hydrogen atom spectra, not even one line can be represented with one eigenvalue.

It is apparent that the simplicity reflected by the change $\{a_j\, b_k\} \rightarrow \{\{a'_j b'_k\}\}$ is due to basis completeness. The problem now is to show if this transformation actually allows us to define a measurement process that, while retaining the unitary time evolution of the whole system, would permit a definition of wave functions for both the measuring apparatus and for the measured system. The interaction pattern induced by the measured object wave function is registered, as it were, in the resulting recording $\{b_k(t)\}$. While the von Neumann operator puts the object system in a basis state, the present model in contrast puts this system in a new well-defined linear superposition. The set of relative phases is conserved.

The final result elicited by the relation (9.2.6) can be written in a self-evident manner as

$$|\psi(t)\rangle - |\psi(t_o)\rangle = \Sigma_j\Sigma_k \{a_j(t)\, b_k(t)\}\, |j^S\rangle|k^D\rangle - \Sigma_j\Sigma_k \{a_j(t_o)\, b_k(t_o)\}\, |j^S\rangle|k^D\rangle = \Sigma_j\Sigma_k \{ a_j(t)\, b_k(t) - a_j(t_o)\, b_k(t_o) \}\, |j^S\rangle|k^D\rangle \quad (9.2.9)$$

This is an exact result. Let us work out now a simple example (in the spirit of the linear response approach).

Consider state k' with amplitude $b_{k\neq k'}(t_o)=0$ and $a_{j\neq j'}(t_o)=0$. The root states are label with k' and j' respectively. Now, let us take in the spectra of the measuring device an energy level k and for the measured system level j such that the transition k' to k is compensated by the transition j to j'. This means that the global system can end up in a state $a_{j'}(t)\, b_k(t)\, |j^S\rangle|k^D\rangle$ after having interacted from state $a_j(t_o)\, b_{k'}(t_o)\, |j^S\rangle|k^D\rangle$; by construction, energy is conserved at the Fence. Thus, it is sufficient to probe the state $b_k(t)\, |k^D\rangle$ to identify the k-level and since we know the initial j'-level the information about the j-level follows. Because we know the spectra for both systems, action can be taken to restore the quantum state of the measured system if you wanted it. A similar action could be taken with the measuring apparatus.

The basic principle of measurement is hence realized at the Fence. If you keep tinkering in Hilbert space, the most extravagant conclusions could be achieved because neither energy conservation, nor other symmetry properties could naturally be taken into account.

The matrix elements of the evolution operator can be cast in terms of the dipole-dipole approximation: $\{a_j(t_f)\ b_k(t_f)\}$.

$$U^{kk'}_{jj'}(t,t_o) = \delta_{jj'}\ \delta_{kk'} - (i)\int_{t_o}^{t_f} dt' \ \{(1/2)\ D_{kk'}(t')\cdot E^{S}_{jj'}(t')) + (1/2)\ E^{D}_{kk'}(t')\cdot S_{jj'}(t')\} + \ldots \qquad (9.2.10)$$

$S_{jj'}(t)$ and $D_{kk'}(t)$ are matrix elements of the transition moment operators, while $E^{D}_{kk'}(t)$ and $E^{S}_{jj'}(t)$ are the matrix elements of the field operators, all operators are in the interaction representation. The matrix elements $D_{k'k}(t)$ are products of time independent matrix elements $D_{kk'} = \langle k|\mu_D|k'\rangle$ and the phase factor $\exp(i(E_k-E_{k'})t/\hbar)$, with an analogous expression for $S_{j'j}(t)$.
The correlation function $F(t,t_o)$ is defined as:

$$C(t,t_o) = \langle\psi(t_o)|\ (|\psi(t)\rangle - |\psi(t_o)\rangle\ ) = \langle\psi(t_o)|\psi(t)\rangle - 1 \qquad (9.2.11)$$

Calculating this quantum overlap function with the initial quantum state, one gets a sum of two amplitudes, named below as C(D[S]) (i.e. a complex number related to changes in the detector quantum state after interaction with the system) and C(S[D]) (i.e. a complex number related to changes in the system wave function after interaction with the detector). For the particular model, the amplitudes C(D[S]) and C(S[D]) are defined to first order by equations (10) and (11),

$$C(D[S]) = \langle\phi_{(D)}|D[S]\rangle = (-i/2)\ \Sigma_k\ \Sigma_{k'}\ b_k^*\ b_{k'}\ (\Sigma_j\ \Sigma_{j'}\ a_j^*\ a_{j'} \int_{t_o}^{t_f} dt'\ E^{S}_{j'j}(t')\cdot D_{k'k}(t')) = \Sigma_k\ b_k^*(t_o)\ b'_k(t_f) \qquad (9.2.12)$$

This equation shows that the object wave function is imprinted in the b'_k coefficients. The new wave function for the measured system reads:

$$C(S[D]) = \langle\varphi_{(S)}|S[D]\rangle = (-i/2)\ \Sigma_j\ a_j^*\ \Sigma_{j'}\ a_{j'} \times (\Sigma_k\ \Sigma_{k'}\ b_k^*\ b_{k'} \int_{t_o}^{t_f} dt'\ E^{D}_{k'k}(t')\cdot S_{j'j}(t')) = \Sigma_j\ a_j^*(t_o)\ a'_j(t_f) \qquad (9.2.13)$$

The measuring device put the imprint on the wave function via the final coefficient a'_j. Equation (10) and (11) above are valid until time t_f, which is the time when the interaction is completed, the Hilbert space evolution is broken at this point. Note that the time integral contains the term $\exp\{i(E_j^S - E_{j'}^S + E_k^D - E_{k'}^D)\, t'\}$. This factor ensures energy conservation. Under resonance conditions, namely, $E_j^S - E_{j'}^S + E_k^D - E_{k'}^D$ equals zero; the contributions from the integral increase proportional to the interaction time interval, $t_f - t_o$. For those combinations of levels that are far off resonance, the integrand oscillates too fast leading to zero contributions.

The correlation function defined in eq.(9.2.9) probes the time evolution of the non-zero initial amplitudes, namely the complete initial quantum state of the system S. This process requires full information that, in most cases, is not easy to gather or is not necessary for the purposes of the experiment you want to carry on. A less detailed correlation function that probes the amplitude at some particular base state, say $|n^S\rangle$, is defined as the modulus square of the projection $\langle n^S|\psi(t)\rangle$:

$$C_{nS}(t) = |\langle n^S|\psi(t)\rangle|^2 = |a_{nS}(t)|^2\, \Sigma_k\, b_k^*(t)\, b_k(t) = |a_{nS}(t)|^2 \qquad (9.2.14)$$

To get the last equality implies the quantum state of the measuring system to be normalized to one at the time an experiment is performed to measure $|a_{nS}(t)|^2$.

Consider sets of laser pulses peaked around definite frequencies. One is used to prepare the quantum state of the system of interest a series of other pulses is used to probe the time evolution via $|a_{nS}(\tau)|^2$. The preparation is done at time t_o; after a time lag τ another laser pulse tuned to a spectral response of the root state $|n^S\rangle$ is sent and the spectral response measured in intensity. According to the interpretation given in Ch.2, $C_{nS}(\tau)$ describes the temporal change of the relative intensity for the spectral response from the n^S–root base state. This is a typical pump-probe experiment.

After measured and measuring systems are parted, a separation of $\{a'_j b'_k\}$ into the sets $\{a'_j\}$ and $\{b'_k\}$ is the natural procedure. The sets of coefficients characterize the quantum state of the respective subsystems. This follows from the structure of equation (10) and (11) above. This result agrees with the cluster decomposition hypothesis commonly used in quantum field theory [19]. One define the functions $|D[S]\rangle$ and $|S[D]\rangle$ after the time lapse t_f-t_o by the new set of amplitudes

$$|D[S]\rangle = \Sigma_k (-i/2) \int_{t_o}^{t_f} dt' \,(\Sigma_j \Sigma_{j'}\, a_j^*\, a_{j'}\, E^S_{j'j}(t))\cdot (\Sigma_{k'}\, b_{k'}\, D_{k'k}(t'))|k^D\rangle = \Sigma_k\, b'_k|k^D\rangle \qquad (9.2.15)$$

$$|S[D]\rangle = \Sigma_j (-i/2) \int_{t_o}^{t_f} dt' \,(\Sigma_k \Sigma_{k'}\, b_k^*\, b_{k'}\, E^D_{k'k}(t'))\cdot (\Sigma_{j'}\, a_{j'}\, S_{j'j}(t'))\, |j^S\rangle = \Sigma_j\, a'_j|j^S\rangle \qquad (9.2.16)$$

The detector term (Cf. D[S]) represents the recording on the measuring apparatus of the interaction with the measured system. The detector contains now all allowed transitions produced by the fields of the object system and nothing else. In fact, as eq. (9.2.15) shows, it contains the complete imprint of the object wave function as prepared *before* interaction via the terms $\Sigma_j\Sigma_{j'}$ a_j* $a_{j'}$ $E_{j'j}^{S}(t)$. The changes produced by interactions with the system have no intrinsic randomness. Moreover, each one of the b'_k terms embodies the interactions with the incoming S-system state; this is a *holographic* type of interaction.

The quantum state of the object system, after interaction, has imprinted (see S[D]) the measuring device wave function via terms, $\Sigma_k \Sigma_{k'}$ b_k* $b_{k'}$ $E_{k'k}^{D}(t')$. This is the term that modulates the object wave function after interaction. Here, again, everything is under control. At this stage, where one considers one system wave function, there are no random effects related to the interaction within this model.

The quantum amplitudes in Hilbert space, obtained from eq. (9.2.5) involve states ranging over the complete energy space. Now, if we want to come back to real space-time, energy conservation is to be imposed thereby eliminating those amplitudes on base states that cannot be "populated" with the available energy. The constraint is introduced in the next section. Note, however, that the amplitudes are modified in Hilbert space where thermodynamics arguments do not apply.

So far energy conservation is concerned, the present model can be made energy conserving. In an energy basis, each basis element $|j^S>|k^D>$ is asymptotically related to the energy label $E_j^S+E_k^D$, and for the set $\{a_jb_k\}$ the energy of the combined state before interaction is $E(\{a_jb_k\})$ taken as equal to the expectation value of H_o. After the systems become asymptotic again the energy $E(\{a_j'b_k'\})$ is the expectation value of H_o over the wave functions given by the eqs. (9.2.14) and (9.2.15) recast in terms of the final amplitudes ($\{a_j'b_k'\}$). For the global system, energy is conserved. The following two relationships hold for the energy partitioning: $\Delta E_S = \Sigma_j(f(a'_j)-f(a_j))E_j^S$ and $\Delta E_D = \Sigma_k(f(b'_k)-f(b_k))E_k^D$, where $f(b_k)$, for instance, is $|b_k|^2$. These deltas represent the change in energy undergone by each subsystem after the interaction. The total energy change is $\Delta E_S+\Delta E_D= 0$. As a result of the measuring interactions, the *changes in the measured system are correlated to the changes in the recording apparatus*. What really matters, in so far as measuring is concerned, is the energy balance: $\Delta E_S+ \Delta E_D = 0$. If there were no energy reshuffling among subsystem states, there would be no measurable effect.

Total linear momentum is also conserved for the present model. Thus, even if the energy "reshuffling" is such that no energy exchange can be detected, the linear momenta of the measuring and measured systems may change while conserving the total momentum. Elastic scattering is a prototype for such exchanges.

In conclusion to this section, in view of the initial conditions imposed to the system, it is apparent that a measuring of one subsystem by another is possible after they have interacted. The problem now is to amplify the effects by using other layers of measuring devices giving signals on the laboratory level. We will discuss some aspects to this problem below.

9.3. Measuring wave functions

The definition retained for a quantum detector is well adapted to a classical view. The effective quantum state the detector is left, reflects all amplitudes the measured system was prepared in, Cf.eq.(9.2.14). The state *reflects a past unitary evolution*, which is totally different from the standard Copenhagen view. We have explicitly stated that entangled states do not make part of the base states.

What about entangled states? These states are those quantum mechanics can show that are fundamentally different from classical physical world. In the following section we continue the analysis of Scully et al. paper that allows us to come back to entanglement.

9.3.1. Measurements including entangled states

One enters in mine fields whenever complementarity arguments are discussed. Those are the historic remnants of early days quantum mechanics. The particle view inserted in descriptions of phenomena leads to most weird aspects because one is mixing two non-commensurable spaces.

Let us stick to the quantum state description we have introduced in our present view from the Fence and pursue the analysis of Scully's et al. paper including now cavity states that are necessary when the physical devices are put before the second double slit screen. After the first screen the beams are collimated by sets of wider slits. The point is that the quantum state is the same for both beams; they are prepared in the same manner. A laser beam is set up perpendicular to the propagation direction from the slits in their way to two cavities that have no interaction among themselves; the beams continue propagate until the end of the cavity and out to interact further away with the second two-slit screen.

The key is to calculate the beam quantum states as they reach the second screen. We know that if they are equal in all respects there will be an interference pattern. If they come out in different quantum states due to the interaction with each cavity then, depending on the degree of difference, the interference pattern will fade away (See Figure 3 of Scully et al.).

Before coming to the quantum state generated by the laser observe the quantum states associated to the beams. They correspond to |Hole-1> and |Hole-2> states in their linear superposition that we write again:

$$|\Phi\rangle = C_1\, |\text{Hole-1}\rangle + C_2\, |\text{Hole-2}\rangle$$

where C_1 = <Hole-1|Φ> and C_2 = <Hole-2|Φ>. For the space amplitude (wave function) multiplying the I-frame quantum state |Ψ>:

$$<x,y,z|(x>x_S)> |\Psi>= \{C_1 \exp(i\gamma_1) <x,y|1> \exp(i(k_x x + k_{y1}(y-D)) + C_2 \exp(i\gamma_2) <x,y|2> \times \exp(i(k_x x + k_{y2}(y+D))\}|\Psi> \quad (9.3.1.1)$$

At the region covered by the laser there is an interaction so that the excited state associated to the internal part of the I-frame system |Ψ*> is assumed to have a sufficiently long lifetime. Thus, at the entrance of the cavities one has the quantum state K:

$$K = \{C_1 \exp(i\gamma_1) <x,y|1> \exp(i(k_x x+ k_{y1}(y-D)) + C_2 \exp(i\gamma_2) <x,y|2> \exp(i(k_x x + k_{y2}(y+D))\}\ C(\Psi^*)\ |\Psi\ ^*> + \{C_1 \exp(i\gamma_1) <x,y|1> \exp(i(k_x x+ k_{y1}(y-D)) + C_2 \exp(i\gamma_2) <x,y|2> \exp(i(k_x x + k_{y2}(y+D))\}\ C(\Psi)\ |\Psi> \quad (9.3.1.2)$$

We take then $C(\Psi^*)\approx 1$ and $C(\Psi)\approx 0$. But *you cannot erase the base states*!

Because we want to tinker with two independent cavities, rearrange the quantum state to make appear the beams:

$$K = C_1 \exp(i\gamma_1) <x,y|1> \exp(i(k_x x+ k_{y1}(y-D)) \{C(\Psi^*)\ |\Psi\ ^*> + C(\Psi)\ |\Psi>\}+ C_2 \exp(i\gamma_2) <x,y|2> \exp(i(k_x x + k_{y2}(y+D)) \times\{C(\Psi^*)\ |\Psi\ ^*> + C(\Psi)\ |\Psi>\} \quad (9.3.1.3)$$

For real space separated cavities the quantum state along the beams can be separately coupled in the following manner:

$$\text{Beam-1: } [|\Psi>\ |\Psi\ ^*>] \rightarrow [|\Psi>|1_1 0_2>\ \ |\Psi^*>|0_1 0_2>]$$
$$\text{Beam-2: } [|\Psi>\ |\Psi\ ^*>] \rightarrow [|\Psi>|0_1 1_2>\ \ |\Psi^*>|0_1 0_2>]$$

The base state $|\Psi>|1_1 0_2>$ corresponds to one photon at cavity 1, zero at cavity 2; the energy is available in the photon field. The base state $|\Psi^*>|0_1 0_2>$ corresponds to an excited state internal I-frame system coupled to the electromagnetic vacuum. Both states have the same energy and can be coupled easily by an EM field of frequency in a neighborhood of $|1_1 0_2>$. The corresponding quantum state is an entangled state.

In order to make contact with Scully's paper, the quantum states are decomposed along beam representations. The quantum state takes on the form:

$$<x,y,z|\text{Beam-1}> = C_1 \exp(i\gamma_1) <x,y|1>\times$$
$$\exp(i(k_x x + k_{y1}(y-D)) \ (C(\Psi;1_1 0_2) \quad C(\Psi^*;0_1 0_2>))$$
$$\times [|\Psi>|1_1 0_2> \quad |\Psi^*>|0_1 0_2>]$$
$$<x,y,z|\text{Beam-2}> = C_2 \exp(i\gamma_2)$$
$$<x,y|2> \exp(i(k_x x + k_{y2}(y+D)) \ (C(\Psi;1_1 0_2) \quad C(\Psi^*;0_1 0_2)$$
$$\times [|\Psi>|0_1 1_2> \quad |\Psi^*>|0_1 0_2>]$$

Remember that $(C(\Psi;1_1 0_2) \ C(\Psi^*;|0_1 0_2>)) \ [|\Psi>|1_1 0_2> \quad |\Psi^*>|0_1 0_2>]$ is a linear superposition (scalar product between row amplitude vector with column base set states. And, in what follows we separate the terms just to simplify the writing.

The interaction with the cavity field put a label onto the base states of the external quantum states. The base sets have one base state element in common: $|\Psi^*>|0_1 0_2>$. Therefore, if we manipulate the system in such a way that the state is defined by $C(\Psi^*;0_1 0_2) = 1$, then $C(\Psi;1_1 0_2)=0$. This is Case 1 with the quantum state impinging at the second screen given by:

$$<x,y,z|\text{Beam-1}> = C_1 \exp(i\gamma_1) <x,y|1> \times$$
$$\exp(i(k_x x + k_{y1}(y-D)) \ |\Psi^*>|0_1 0_2>$$
$$<x,y,z|\text{Beam-2}> = C_2 \exp(i\gamma_2) <x,y|2> \times$$
$$\exp(i(k_x x + k_{y2}(y+D)) \ |\Psi^*>|0_1 0_2>$$

As a matter of quantum mechanical fact, once the beams are prepared in the way described they form a linear superposition:

$$C'_1 \exp(i\gamma_1)<x,y|1>\exp(i(k_x x + k_{y1}(y-D)) +$$
$$C'_2 \exp(i\gamma_2)<x,y|2> \exp(i(k_x x + k_{y2}(y+D))$$

This term multiplies the base state $|\Psi^*>|0_1 0_2>$ that carry the information about the state of the resonance cavities. The conclusion is simple: for this case, interference pattern will show up. And it will be the same if $C'_1 = C_1$ and $C'_2 = C_2$ as if nothing was put between screen one and the recording device.

Case 2 corresponds for example to a photon left at cavity one. The quantum state with $C(\Psi;1_1 0_2)=1$ and $C(\Psi^*;|0_1 0_2>=0$ reads as follows:

$$<x,y,z|\text{Beam-1}> =$$

$$C_1 \exp(i\gamma_1) \langle x,y|1\rangle \exp(i(k_x x + k_{y1}(y-D))) |\Psi\rangle|1_1 0_2\rangle$$

For Beam 2, because the amplitudes of the cavity field are determined by $C(\Psi^*;|0_1 0_2)=1$ one gets:

$$\langle x,y,z|\text{Beam-2}\rangle =$$
$$C_2 \exp(i\gamma_2) \langle x,y|2\rangle \exp(i(k_x x + k_{y2}(y+D))) |\Psi^*\rangle|0_1 0_2\rangle$$

Once the system leaves the cavity we have the quantum state meaning with this statement that theere is no presence in the assigned volume:

$$|\text{ Case 2 }\rangle = C_1 \exp(i\gamma_1) \langle x,y|1\rangle$$
$$\times \exp(i(k_x x + k_{y1}(y-D))) |\Psi\rangle|1_1 0_2\rangle +$$
$$C_2 \exp(i\gamma_2) \langle x,y|2\rangle \times$$
$$\exp(i(k_x x + k_{y2}(y+D))) |\Psi^*\rangle|0_1 0_2\rangle$$

It is apparent that the interference pattern will not show up because the overlap $\langle 1_1 0_2|0_1 0_2\rangle$ is zero. Of course you realize that the interference pattern is still there albeit invisible! Of course, it is invisible to the detecting device, but the quantum state originating the interference is right there. The information has not been erased. The intensity pattern does disappear but the quantum state has it there.

9.3.2. Entanglement

Entangled states cannot be put as simple products. The example found above permits seen the mathematical form:

$$(C(\Psi;1_1 0_2)\quad C(\Psi^*;0_1 0_2))[|\Psi\rangle|1_1 0_2\rangle\quad |\Psi^*\rangle|0_1 0_2\rangle] =$$
$$C(\Psi^*;0_1 0_2)\, |\Psi^*\rangle|0_1 0_2\rangle + C(\Psi;1_1 0_2)\, |\Psi\rangle|1_1 0_2\rangle =$$
$$|\text{Entangled}\rangle$$

The stationary state is characterized by $C(\Psi^*;0_1 0_2)= C(\Psi;1_1 0_2)= 1/\sqrt{2}$. For this state, the electromagnetic field never has the total energy used to excite the ground state. A time independent situation would produce an infinite lifetime. However, any frequency ω in the vicinity of the resonance frequency ω show a time dependent quantum states ($\cos \omega t \quad \sin \omega t$) and its orthogonal companion ($\cos \omega t$ - $\sin \omega t$); this state is normalized to one, at all times.

The writing of $|\Psi^*\rangle$ implies that for a particular material system, the lifetime of the excited state is sufficiently long to allow for experimental manipulations at the Fence.

Now return to Case 2 above. The base state $|\Psi^*\rangle|0_1 0_2\rangle$ had an amplitude equal to zero so that the entangled state is: (1 0) $[|\Psi\rangle|1_1 0_2\rangle \ \ |\Psi^*\rangle|0_1 0_2\rangle]$. The photon appears as if it were at cavity 1. What do happen if you design an experimental device that the quantum state is rotated into (0 1)-state?

Well. You have "erased" the response function at cavity 1 and constructed the quantum state of Case 1! The material support of the interference pattern has not been touched thereby implying that the Case-1 pattern emerges again. In fact, it was always there. And you can play around because of the entanglement between the internal quantum system and the photon field. You can delay as much as you want provided the entangled state is still there! Tinkering in a similar manner with cavity 2 one will get the same results.

The only thing you have to be able to calculate is the quantum state entering the scattering centers (slits) and the characteristics of the interaction. Quantum mechanics is about quantum states not about quantum particles or waves. The complementarity principle makes sense when you insist introducing (quasi) classical concepts of particles, waves and paths. If you do not need them, as we show here, then that principle is no longer useful.

9.3.3. Einstein-Podolski-Rosen thought experiment

The base states necessary for a discussion of the famous EPR-experiment can be reduced to three generic ones. A two-label system with total spin $S=s_1+s_2 = 1$ with zero eigen value along the 3-direction (arbitrary): $s_{z1}+s_{z2} = 0$. The quantum state projected in configuration space with base ket $|\ \mathbf{x}_1,\mathbf{x}_2\rangle$ is a symmetric space function $\phi_\Sigma(\mathbf{x}_1,\mathbf{x}_2) = \langle \mathbf{x}_1,\mathbf{x}_2|\phi_\Sigma\rangle$. There are two 1-systems with base sets: $\phi_1(\mathbf{x})$ and $\phi_2(\mathbf{x}')$ each one associated to a spin 1/2 spinor, they are called asymptotic systems. The physical process consists in a transition where the amplitude at the complex system base state changes into amplitude (equal one) for the asymptotic systems. Thus, from one I-frame we go into a two I-frame systems in real space. Let us designate the momentum associated to I-frame one by $\mathbf{k}_1$ and $\mathbf{k}_2$ for the other. The simplest procedure is to take the origin of the global I-frame so that $\mathbf{P}=\mathbf{0}$ and linear momentum conservation forces $\mathbf{k}_1=-\mathbf{k}_2$. At the antipodes, $\mathbf{k}_1\cdot\mathbf{x}=-\mathbf{k}_2\cdot(-\mathbf{x})$ and the plane wave states are equal there. The spin quantum state for I-frame 1 corresponds to the linear superposition $(c_1\ c_2)[\alpha\ \ \beta\ ]$ while the second I-frame system should have $(c_2\ -c_1)[\alpha\ \ \beta]$, namely an orthogonal quantum state.

There are two generic situations that would permit a simple discussion of EPR case. First, we do know that the 1-system has a well-defined quantum state and the asymptotic states are necessarily correlated by linear momentum conservation. What about spin measurement? In this case, any measurement telling the experimenter the quantum state is $(c_1\ c_2)$ (I-frame 1) the person knows that a measurement carried out simultaneously for I-frame 2 would yield $(c_2\ -c_1)$. There is no signal to be sent! Just the knowledge that will permit to prepare the state (c_2 $-c_1$) at I-frame 2 because you knew the state (c_1 c_2) at I-frame 1. The systems were initially correlated. That is all.

Second, you do not know about the correlation. If you measure the quantum state of a system that happen to pass via your measuring device, the result being (c_2 $-c_1$), then that's it.

Now, if you keep measuring the system prepared many times and the source happen to be of the kind we discussed above, then the experimenter will measure either by the row vector (c_2 $-c_1$) or (c_1 c_2) until this person will realize that there is a correlation source at the origin.

The EPR-case just dissolves into nothing more than a misunderstanding. As it is well acknowledged today, EPR premise is a sufficient condition; it is not a necessary one.

9.3.4. Delayed choice experiments

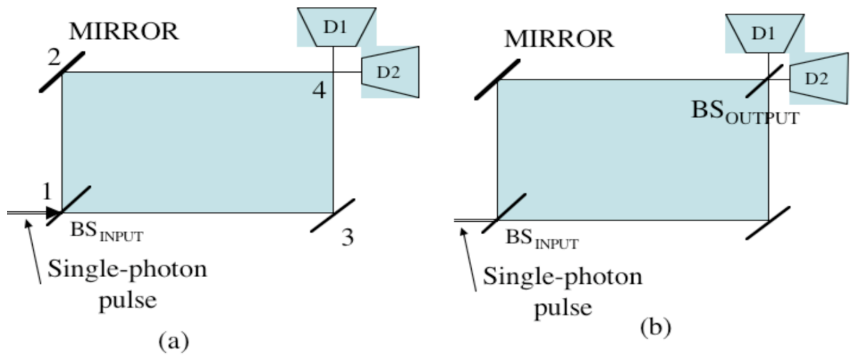
During the "thought" experiment era, Wheeler proposed a challenging one christened by him as a delayed choice experiment (J.A.Wheeler in Quantum Theory and Measurement, edited by J.A.Wheeler and W.H.Zurek, Princeton University Press, NJ, 1984; pp. 182-213). Today, families of experimental setups can be constructed with beam-splitters, mirrors and photo-detectors so that the ideas underlying "thought" experiment could be tested. Recently, it was reported an experimental realization of Wheeler's delayed-choice thought ("gedanken") experiment (V. Jacques et al. Science (2007) 315, 966-968); so far results descriptions are made with the help of particle paradigm. In what follows, a discussion in terms of quantum states is introduced. A comparison of both approaches is given at the end of this section.

The setup mixes classical elements distributed in real space with quantum mechanical "functionalities" proper to a Hilbert space representation. We define first the basic elements of the model and thereafter construct wave functions in quantum mechanical terms; finally, a quantized EM-field representation is used to further discuss the experimental results:

1. Beam splitter (BS). Given a quantum state $|\Psi\rangle$ impinging on a BS, after interacting there the quantum state related to vertex-1 is given by a linear superposition:

$$|1\rangle=\langle 1a|\Psi\rangle\ |1a\rangle + i\langle 1b|\Psi\rangle|1b\rangle$$

Thus, there is a transition from the quantum state $|\Psi\rangle$ to $|1\rangle$. Note that in the Schemes below this quantum state relates to a single-photon pulse in real space.



The base state |1a> is directed along vertices 1-3, sharing the direction of the initial photon pulse state; base state |1b> (direction 1-2) is orthogonal to |1a> and affected by a phase factor $\exp(i\pi/2) = i$ to signal this change. For a beam-splitter acting as a half-silvered mirror the amplitudes take on values $|\langle 1a|\Psi\rangle| = |\langle 1b|\Psi\rangle| = 1/\sqrt{2}$. These numbers signal the way you have prepared the beam splitter, i.e. a perfectly balanced BS.

2. Mirror (M) at vertex-2, oriented as if it were parallel translated with respect to the BS_{inp}, takes quantum state |1> into quantum state |2> on the base state [|2a> |2b>]:

$$|2\rangle = |2a\rangle\langle 2a|1\rangle + i\langle 2b|1\rangle\,|2b\rangle =$$
$$(i\langle 2a|1\rangle \;\; \langle 2b|1\rangle)\,[|2a\rangle \;\; |2b\rangle]$$

In the particular case signaled in Scheme (a) we have $|\langle 2b|1\rangle| = 0$ because the mirror, in real space, changes the beam direction only. The quantum state |2> maps onto (i 0) [|2a> |2b>].

3. Mirror at vertex-3. The quantum state is given in the local base set as:

$$|3\rangle = |3a\rangle\langle 3a|1\rangle + i\langle 3b|1\rangle\,|3b\rangle$$

The amplitude <3a|1> must be zero due to the physical property of this mirror. The quantum state |3> looks like (0 i) [|3a> |3b>].

4. Quantum states |2> and |3> propagate toward vertex-4 each from a different direction. At vertex-4 there is no material system to interact either with quantum state |2> or |3>, thereby implying that the detector D1 will interact with quantum state |3>, and detector D2 would do it with the quantum state |2>.

Thus, detector D1 will sense a response controlled by the amplitude at |3b> while D2 will sense a response controlled by the amplitude at |2a>. It remains to calculate these amplitudes as a function of the input state. This is done E&E-9.3-x below.

E&E-9.3-x Calculate states |2> and |3> as a function of input data for case (a)
Replace in the generic definitions of these quantum states the ket obtained at vertex-1, |1>

$$|2\rangle = |2a\rangle\langle 2a|1\rangle + i\langle 2b|1\rangle\,|2b\rangle = |2a\rangle\langle 2a|1\rangle + 0\,|2b\rangle =$$
$$|2a\rangle\langle 2a|\{\langle 1a|\Psi\rangle\,|1a\rangle + i\langle 1b|\Psi\rangle|1b\rangle\} + 0|2b\rangle$$

Do the algebra and bear in mind that $\langle 2a|1b\rangle = \langle 2b|1a\rangle$ = zero because in a rigged Hilbert space base vectors for different directions are orthogonal. The amplitudes $\langle 2a|1a\rangle$ and $\langle 2b|1b\rangle$ are delta functions with origin at the crossing points. We simplify the notations and take them equal to 1 there. The result is:

$$|2\rangle = |2a\rangle\,\langle 1a|\Psi\rangle + 0\,|2b\rangle$$

By construction $|\langle 1a|\Psi\rangle|=1/\sqrt{2}$ and the spectral response intensity at detector D2 will be half the intensity of the input pulse.
A similar procedure applies to state |3> to get:

$$|3\rangle = 0\,|2a\rangle + |3b\rangle\, i\langle 1b|\Psi\rangle$$

Here, again, the model we are discussing leads to $|\langle 1b|\Psi\rangle|=1/\sqrt{2}$ and half of the spectral response obtains at detector D1.

The results obtained above are correlated in a typical quantum mechanical sense. For, if something happens to the quantum state related to the beam 1-2, before coming in contact with M-2, namely a change of $\langle 1b|\Psi\rangle$ amplitude due to a change in the properties of BS_{input} this will be echoed via the mirror at vertex-3. Thus it will affect measurements at detector D1 that is found along direction 3-4.

Following Wheeler's spirit, a second BS is required at the cross of directions associated to |2a> and |3b> base states. The setup is depicted in Scheme (b).

The distances 124 and 134 are identical by construction; the time-of-flight is equal for two identical light signals, in real space. The propagation along these paths brings the quantum states at point 4 simultaneously. At the crossing point 4 the input states are mixed by BS_{output}; naming the base state pointing to detector D2 as |5> and |5'> in the direction of D1, and selecting a perfectly balanced BS_{output}, the quantum states after interaction map as follows:

$$|2\rangle \rightarrow (1/\sqrt{2})\,(\,|5\rangle + i\,|5'\rangle)$$
$$|3\rangle \rightarrow (1/\sqrt{2})\,(\,i|5\rangle + |5'\rangle)$$

At vertex 4 we denote the incoming beams by |4a> → |2> and |4b> → |3> to bring harmony with notation by Wheeler. The state |3b> is related to M-3 via:

$$|3b\rangle \rightarrow iR|4a\rangle;$$

|4a> relates to M-1 via:

$$|2a\rangle \rightarrow iR|4b\rangle.$$

The reflection amplitude R for the mirrors is R=1.

Now, after going through BS_{output} there are two equations for beams |4a> and |4b>

$$|4a\rangle \rightarrow (1/\sqrt{2})\,(\,|5\rangle + i\,|5'\rangle)$$

$$|4b\rangle \rightarrow (1/\sqrt{2})\,(\,i|5\rangle + |5'\rangle)$$

Picking up the definitions of base states |2a> and |3b> we form the quantum state in a unified base set:

$$|\Phi\rangle \rightarrow |3b\rangle\, i\langle 1b|\Psi\rangle + |2a\rangle \langle 1a|\Psi\rangle =$$
$$iR|4a\rangle\, i\langle 1b|\Psi\rangle + iR|4b\rangle \langle 1a|\Psi\rangle$$

For the case we had chosen of perfect BS, $|\langle 1b|\Psi\rangle| = |\langle 1a|\Psi\rangle| = 1/\sqrt{2}$, the result is:

$$|\Phi\rangle \rightarrow 1/\sqrt{2}(\, i^2|4a\rangle + i||4b\rangle = 1/\sqrt{2}(\, -|4a\rangle + i|4b\rangle$$

The outgoing states are used now:

$$|\Phi\rangle \rightarrow 1/\sqrt{2}(\, i^2(1/\sqrt{2})\,(|5\rangle + i\,|5'\rangle) + i\,(1/\sqrt{2})\,(i|5\rangle + |5'\rangle)$$

Calculating the sum one gets: $1/\sqrt{2}(\, - (1/\sqrt{2})\,(\,|5\rangle - (1/\sqrt{2})\,|5\rangle - i\,|5'\rangle + i|5'\rangle)$ and the final assignment reads as:

$$|\Phi\rangle \rightarrow = (1/\sqrt{2})\{(-2/\sqrt{2})|5\rangle + 0i|5'\rangle\}$$

The detector put along direction |5'> cannot detect a signal because the amplitude is zero, detector D2 gets all signal.

The delayed choice issue can be stated as follows. For a 1-photon experiment, because it takes a finite lapse of time for the quantum state to propagate and arrive at vertex-4 the question arose whether an experiment for which one delays the positioning of BS_{output} until the last "femtosecond" before arrival will show the same result compared to other one where BS_{output} was there all the time.

This question makes sense if we look first to the result obtained for Scheme (a). For now, there is one and only one event at either detector. The conclusion may be that either the quantum state is given by $C_1|2\rangle$ or $C_3|3\rangle$; the alternatives (linear superpositions) when seen from standard logics are exclusive. You have in this case $C_1=1$ and $C_3=0$, or vice versa. You have jumped to this conclusion for the 1-system case because you detect the energy exchanged at only one detector.

Now comes in the delayed experiment. Because the quantum state "collapses" on to say state $C_1|2\rangle$, and now "knowing" this we put a BS_{output} before the quantum state propagates to vertex-4 the result will be the linear superposition:

$$|2\rangle \rightarrow |5\rangle \langle 5|2\rangle + |5'\rangle\, i\langle 5'|2\rangle \rightarrow$$

$$|5\rangle\, i\langle 5|2a\rangle + |5'\rangle\, i^2\langle 5'|2a\rangle$$

Because base states |2a> and |5'> are orthogonal then <5'|2a>=0 and the result will be a click at D2 and for state |3> the click will be at D1. The final counting of N independent 1-photon samples will be N/2 evenly distributed. This model corresponds to a *collapse* of the wave function. The interaction between the ingoing quantum state with the input BS put the system either with state |3> or with state |2>, the dimension 2 of Hilbert space vanishes. For this model, after interaction with the output BS the system is thrown either on state |5> or |5'>.

The quantum mechanical result, on the other hand, even if you put BS_{output} at the last moment, will be zero-count at D1 and full-count at D2.

9.3.5. Particle picture and delayed choice experiment

The particle picture has not been retained in this book. Yet, it is a dominant view. It seems timely to introduce it via the master presentation made by Wheeler himself.

Referring to the fundamental discussions between Albert Einstein and Nils Bohr, Wheeler writes:

"Of all the idealized experiments taken up by the two friends in their effort to win agreement, none is simpler than the beam splitter". Note that our Schemes (a) and (b) replace Figure 4 from Wheeler paper. He continues: "With the final half-silvered mirror in place the photoreceptor at the" lower left (D2 for the present case) "click-click as the successive photons arrive but the adjacent counter register nothing"; this corresponds to our D1 counter. "This is evidence of interference between beams 4a and 4b; or, in photon language, evidence that each arriving light quantum has arrived by both routes," that is 134 *and* 124. "In such experiments, Einstein originally argued, it is unreasonable for a single photon to travel simultaneously two routes. Remove the half-silvered mirror", (the one called BS_{output}), as in Scheme (a), "one will find that the one counter goes off, or the other. Thus the photon has traveled only *one* route. It travels only one route, but it travels both routes; it travels both routes, but it travels only one route." The text continues: "What a nonsense! How obvious it is that quantum theory is inconsistent!"

So far goes Einstein's criticism. The answer by Bohr as quoted by Wheeler is interesting: " Bohr emphasized that there is no inconsistency. We are dealing with two different experiments. The one with the half-silvered mirror removed tells which route. The one with the half-silvered mirror in place provides evidence that the photon traveled both routes. But it is impossible to do both experiments at once".

It is not difficult to see that real and Hilbert space descriptions are mixed up. Whenever a particle description is introduced to describe quantum mechanical outcomes weirdness pops up.

Quantum mechanics is about quantum states and their time evolution. They are related to material substrates in real space, no doubt, but as such they belong to Hilbert space. In this latter space, pictorial descriptions originated in our real world perceptions do not make sense. There are no quantum entities that can behave like particles or waves; see the comments made by P.Knight in Nature (**395** (1998)12-13). But according to the present approach, they behave as quantum states in Hilbert space. These latter can be diffracted and modulate interference patterns. This latter is taken as the signal of wave behavior in the standard texts. But, as water waves in a pond designed to mimic a two-slit set up, and electromagnetic waves in similar arrangement produce interference patterns, it does not mean that water waves are equivalent or equal to electromagnetic waves. Quantum states in particular situations do relate to patterns of interference. At detection, they exchange energy in quanta; this is a fact inferred by Planck. And as we saw in the two slit experiments discussed in section 9.3.1 as an event realizes at a given spot it is modulated by the wave function there. The "particle" and "wave" behavior are "simultaneously" in effect at the point where the click-event emerges. The complementarity principle applies only to the real concepts of particle and wave. It has no relevance if a full quantum mechanical description is at work.

The flaw in Bohr's view lies in the assumption that the measuring device must be macroscopic and "reality", the one emerging in our daily life world, to be communicated with every day language. But language grows as new "continents" are discovered and new words and concepts would find their way to the civil society beyond the jargon we employ in scientists' societies. Material systems (macroscopic or microscopic) sustain all kinds of quantum states. Perceptions of change are rooted in changes of quantum states. These states are not always reducible to objects. We better get used to these developments instead of using non-sense language to talk about.

9.4 What are Quantum States?

S.Malin in a recent paper (Quantum Inf.Proc. 5, 233(2006)) raised this question and presented strong arguments against the ontic and the epistemic interpretations of quantum states.

Before closing this chapter we present a short discussion on these issues including Malin's interpretation.

To get to the point, let us give three quotations from Malin's paper concerning the ontic viewpoint:

1) "…Einstein pointed out that the collapse, which he assumed took place on a t=const. hypersurface, means that the influence of the appearance of an elementary quantum events leads to an instantaneous (faster that light) propagation of the change in the value of the wave function everywhere on the hypersurface. Einstein tacitly assumed the ontic interpretation of the wave function".
2) "…the ontic interpretation is incompatible with Einstein's principle of covariance."
3) Concerning Schrödinger cat paradox one reads: "According to the ontic interpretation the cat is in a state of superposition of being alive and being dead in various proportions that change as time goes by."

For the epistemic interpretation "the superposition says nothing about the cat, it speaks about us. It says that we don't know how the cat is…"

The third interpretation try to overcome the "knower" by saying the quantum state is the place where the available or potential knowledge about the system is found. Malis propose three principles or working hypothesis:

1) QSs represent knowledge available about the potentialities of a quantum system, knowledge from the perspective of a particular location in space;

2) Collecting all such perspectives is all what there is about a quantum system;
3) "Whether or not the development of entangled systems involves *an apparent* superluminal connection, such connection can never be used for superluminal connection, such connection can never be used for superluminal transmission of information."

The ontic point of view takes the QS to represent the material system itself. The epistemic interpretation denies a relationship with the material system to the extent a QS says something about our knowledge about the system. The QS as perspectives on an available knowledge about potentialities is a step along a direction away physics.

The crux of the problem locates in the particle model pervading these interpretations. As a matter of fact one may ask: do we need an interpretation of quantum states?

9.4.1 Back to "our" quantum states

Go back to the beginning of this chapter and read the allegory there. Now you better take it seriously into account. All the discussions given so far World allow you to get a more clear Picture of what a quantum state might be.

We are in front of a real space state incorporating all information the EM field could grasp from the material system via its quantum state. Sustained by the photon field the quantum state makes an impront that can be translated in space at the speed of Light. The quantum state is broadcast in all directions if sufficient energy is made available.

The quantum state can be copied into a material support appropriate to the systems. Light is one material system able to transport quantum status in full detail or partially. It is not knowledge which is being transported but information. It is not the supporting object which is translated but just the quantum mechanical element we name quantum state.

A quantum state is a physical system in laboratory space.

In Hilbert space, a quantum state is a foundational element in the construction of the theory.

Modern Technology has open the way to manipulate and transport quantum states.

9.5 Final remarks

The replacement of objects (molecules, atoms) by quantum states sustained by the material constituents (electrons, nuclei) is a fundamental result. Quantum mechanics provides the mathematical framework to develop the concept of quantum state. Classical physics permits studying objects. They are certainly complementary in scope.

There people that have difficulties in understanding the difference between a material system and chemical quantum systems.

To sense the difference look at the chemical systems identified by the formulae: H_2+CO corresponding to two asymptotic states in the sense that they can be prepared in the laboratory independently of each other and then prepare crossing molecular beams. At the crossing, the material system is characterized by 16 electrons and the sum of nuclear charges +16. The chemical system: H_2-CO, formic aldehyde is certainly different from the asymptotic system but it has the same material composition: 16 electrons and +16 as the sum of nuclear charges. Of course, you may have the asymptotic system: H+HCO, H+H+CO, etc. Note that H, HCO, CO, etc. refer to substances (objects) that can be “bottled” and shelved in a laboratory. Quantum states referring to the same material system will naturally cover all possible chemistry. But you cannot shelve a quantum state equally easy if they relate to ecgited states. If you cannot see the difference quantum states and substances, I cannot help you.

A given quantum state informs us about the possible relative responses with respect to a fixed base set. Using appropriate electromagnetic radiations we can manipulate it at will. The simutators have open doors to produce new substances and processes if they come to mastering the quantum World.